PART I

MECHANICS

STATICS OF RIGID BODIES

M-1. Magnitude of Forces. The first lecture on statics may well begin with a display of forces of varying magnitudes, best shown by weights of familiar objects. A 35-lb weight or a 16-lb hammer from the track department may be shown. Illustrate "a pint's a pound the world around" with milk, eggs, and vegetables. Mention the English and Canadian 10-lb gallon and the correspondence between the German *pfund* and the demikilo. Compare side by side the metric system of weights and the pound-ounce-grain system. Arrange a series of springs of varying stiffness to which may be attached the weights previously discussed so that the relation between stretch and magnitude of force will be clearly appreciated. Finally, show a slide, or display a diagram of typical masses:

Typical mass	Weight, g	Typical mass	Weight, g
Universe	10^{50}	Man	10^{5}
Star cluster	10^{40}	Common pin	10^{-1}
Sun	10^{33}	Pencil mark	10^{-6}
Earth	10^{28}	Speck of dust	10^{-12}
Moon	10^{26}	Heavy molecule	10^{-21}
Mount Everest	10^{18}	Hydrogen atom	10^{-24}
Pyramid of Cheops	10^{11}	Electron	10^{-27}

M-2. Error in Judgment of Weight. Several pairs of specimens, consisting of a pine block 2 by 2 by 10 in. and a brass tube 1.3 in. in diameter and 1.5 in. long, filled with lead, are passed about the class. The wood should be adjusted in length so that it is about 2 or 3 g lighter than the metal. The class will, however, almost unanimously pronounce the wood heavier. The error may be shown readily with an ordinary balance. The effect is heightened if a large piece of balsa wood, whose density is about one-third that of pine, is compared with an iron weight slightly heavier than the balsa.

M-3. Components of a Vector. Two screens are set at an angle of 90° to each other and facing the class. Between them is placed a horizontal wooden arrow 5 or 6 in. long, to represent a vector. Two projection lanterns are used to cast shadows of the arrow normally on the two screens. As the arrow is rotated by a vertical wire spindle, the class can watch one component grow as the other decreases; when one component is zero, the other is equal to the true magnitude of the vector and is parallel thereto. For generality, a third light may be added, if desired, to give the third component of a vector in a plane perpendicular to the other two.

M-4. Resolution of Forces. A spring balance S_1 with a large dial is supported from a rigid vertical rod so that it may be pulled horizontally (Fig. 1). A string from it passes to a ring R that is kept from falling by a peg P in a vertical support. A second string leads from the ring to a nail N at the bottom of the upright. The third force is provided by another spring balance S_2. The direction and magnitude of this force are varied until equilibrium is established, whereupon the peg can be removed without causing a change in the position of the ring. By changing the position of the first balance, various combinations of forces may be produced.

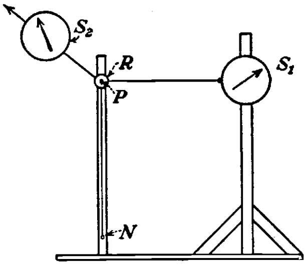


Fig. 1.—Resolution of forces.

M-5. A heavy weight (20 to 30 lb) is suspended by a rope from the ceiling or from a sturdy wall hook so as to form a pendulum 10 to 20 ft long. The weight is displaced from its equilibrium position by a horizontal force whose magnitude (1 to 2 lb per ft of displacement) is shown by a spring balance.

M-6. Sailboat. A large photographic tray or similar shallow container not less than 2 ft square is filled to the brim with water. An electric fan is set up 6 or 8 ft distant so as to blow a uniform breeze across the water surface. A boat (Fig. 2) is made of a 3-in. cork stopper; a thin strip of metal is inserted in a diametral saw cut to serve as a keel. The simple resolution of the force due to wind pressure on the cork alone is illustrated by watching the motion when the keel is pointed in various directions with respect to the wind. The double resolution of force is shown by erecting a sail (a calling card supported with a pin) on the hull. By proper adjustment of the angle between the plane of the keel and the plane of the sail, the boat can be made to sail close to the wind. An enlarged blackboard diagram or a projected slide showing the forces involved should accompany the demonstration.

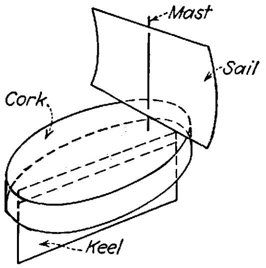


FIG. 2.—Simple sailboat as an example of the resolution of forces.

M-7. Resolution of Forces. This experiment demonstrates the resolution of a vertical force into many oblique forces, each having a vertical and a horizontal component. A collar is soldered to a 1-ft length of ½-in. brass tubing, and the tube is supported vertically from a strong stand (Fig. 3). A ⅜-in. wooden dowel is adjusted to slide loosely in the tube. Two or three thicknesses of soft tissue paper are fastened over the lower end of the tube with a rubber band, and fine dry sand is poured into the tube to a depth of 2 or 3 in. The dowel rod is then introduced, and a wooden block with a ⅜-in. hole bored part way through it is placed on top of the rod. Weights are applied to the block until the tissue breaks. Fifty pounds will be supported indefinitely, and a much greater weight may be used before the paper breaks. Most of the weight is of course supported by the vertical components of the forces between the tube and the particles of sand next it. It may be shown that without the sand the paper alone will support only a very small weight.

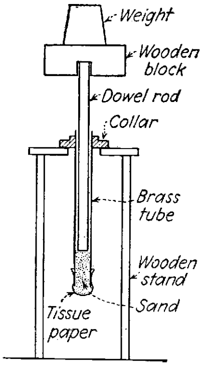


FIG. 3.—Resolution of forces. A heavy weight is supported by the sand, although only a thin tissue keeps the sand in the tube.

M-8. Addition of Forces. A cord is attached to each end of a series of three or four spring balances connected hook to ring and laid on the table. The cords run over pulleys at opposite ends of the table, and a 10-lb weight is hung on each cord. The students are asked to decide what the readings of the balances should be. In most cases, they are surprised to discover that all the balances read the same and that this reading is 10 lb and not 20 lb.

M-9. A heavy plank (say 25 lb) is hung horizontally by means of three spring balances hooked into screw eyes uniformly distributed along the top edge. Each balance reads one-third the weight of the plank. The three balances are then arranged in tandem, the lower one hooking into the center screw eye. Each balance reads, except for the weight of the balances below it, the total weight of the plank. Finally, two balances are hooked to the outside screw eyes, and the third to an eye in the center of a light bar passing through the rings of the first two balances. This balance then reads the full weight, the other two one-half each. If a heavy plank is used, the weights of the balances are more nearly negligible, and the distribution of forces throughout the array is more obvious.

M-10. Equilibrium of Two Forces. A student is asked to lift a heavy weight (50 to 100 lb) with a spring balance. The weight is given by the reading of the balance. The weight is removed, and another student then pulls horizontally on the hook of the balance while the first holds the ring. Grips similar to those used for a 16-lb track hammer are convenient. Although each student pulls with the same exertion as when lifting the weight, the reading of the balance is the same as before. The single balance is then replaced by two in tandem, and each then reads the force previously given by the single balance. The forces involved must be large compared with the weights of the balances.

M-11. Equilibrium of Three Forces. Two spring balances, with large dials for visibility, are attached to a 4-ft length of cord, from the middle of which a 20-lb weight is hung. Two students hold the balances so that the strings are vertical; the balance readings are each 10 lb. If the students then move apart, so that the angle between the strings increases, the readings of the balances increase but remain equal to one another. With the aid of a large protractor the angles may be determined, and from them and the known value of the weight the tensions in the cords may be calculated and compared with the balance readings, and their horizontal and vertical components may be computed.

M-12. A circular board, mounted with its plane vertical, carries a peg at its center and three or more pulleys around its rim. Strings passing over the pulleys are attached to a ring fitting loosely over the central peg. Weights are hung from each string, and their magnitudes and the positions of the pulleys are adjusted until the ring is free of the peg, which may then be removed. The forces are now in equilibrium. They may be represented to scale by lines drawn on the blackboard parallel to the strings. Application of the parallelogram method to this diagram shows that the resultant force is zero; or the three vectors representing the forces form a closed triangle.

M-13. Two spring balances are hung from nails at opposite ends of the blackboard and are connected by a string, from the middle of which weights are hung. Lines are drawn on the blackboard under the strings, and lengths are laid off on them proportional to the forces. The resulting lines may then be moved parallel to themselves so as to form a closed triangle (zero resultant force); or if two of the lines are made the sides of a parallelogram, its diagonal, which represents the resultant of the two forces, will be found equal in length and opposite in sense to the third force, the equilibrant. If the string supporting the weights is *knotted* to the string connected to the hooks of the two spring balances at some point other than the center of this string, then in general no two of the forces involved will be equal.

The experiment is capable of many variations. A third spring balance may be used to apply a force near the middle of the string, and this force need not then be vertical. All the forces may be produced by weights if pulleys are fastened to the top or to the sides of the blackboard, and the spring balances are then eliminated.

M-14. Another method is to use a square wooden frame equipped with a number of screw hooks projecting inward. The frame is hung against the lecture-room blackboard, and a system consisting of several light spring balances attached by cords to a ring is stretched out within the framework so that the directions and magnitudes of the several forces applied to the common point have any desired value. The directions of the cords are then recorded by ruling along them on the blackboard with chalk. The frame is removed from the board and a force diagram completed upon the basis of the recorded chalk lines.

M-15. Inexpensive Force Apparatus. The spring balances of M-14 may be replaced with long closely wound screen-door springs, four or more of which are fastened to a common ring.

The springs may be simply calibrated with known weights so that by the aid of the straight-line graph of Hooke's law the tension in any spring may be found in terms of its length. The frame may be provided with a number of holes into which violin pegs are fitted. In this way, the length of a given cord attached to a spring and hence the directions of other forces may be varied. Vector diagrams may be quickly drawn upon the blackboard by indicating the position of the ring, the direction of each spring, and its length. If springs of equal length and stiffness are used, then each force vector will be proportional to the *extension* of its corresponding spring (but not proportional to the total length).

M-16. Components of Force. Two pieces of wood, each 1 by 3 in. and 1 ft long, are hinged together (Fig. 4). An 18-in. length of stout rope is fastened to two heavy screw eyes in two blocks of wood provided with notches. When the ends of the hinged pieces are placed in the notches and the hinged joint is loaded, the rope may be broken by the application of a much smaller force than would be required in the case of a direct pull.

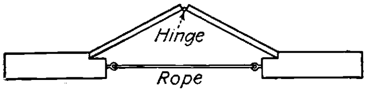


Fig. 4.—Components of force. A small force applied vertically downward at the hinged joint will break the rope.

M-17. The method of swinging on the halyards employed in hoisting a sail may be demonstrated. A rope passes over a pulley in the ceiling to a weight sufficiently heavy so that no one can lift it directly (Fig. 5). The rope passes under a second pulley attached to the floor beneath the first and thence to a cleat. One student pulls horizontally on the vertical rope between the pulleys and after he has drawn it aside 6 in. or 1 ft suddenly releases it. A second student takes a turn around the cleat when the first pulls and takes up the slack quickly when the strain is released. Two light students can lift a weight in this way that the heaviest man in the class cannot move by direct pull. Rope at least $\frac{1}{2}$ in. in diameter should be used.

M-18. Car on Inclined Plane. A small car is held on an inclined plane by a wooden block under its wheels. A string parallel to the plane passes from the car over an independently supported pulley at the top of the incline to a weight holder, where by the addition of weights the component of gravity parallel to the plane is balanced (Fig. 6). The block may then be removed. A second string attached to the car at its center of gravity passes perpendicular to the plane over another pulley to another weight holder, to which weights are added until the pressure of the car on the plane is reduced to zero. The plane may then be removed.

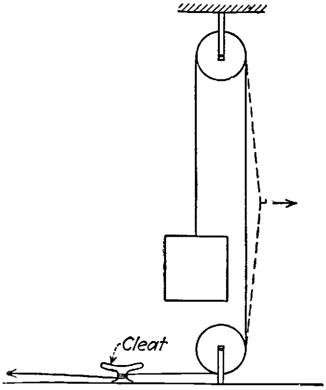


FIG. 5.—Swinging on the halyards. The heavy weight may be lifted by the application of successive small horizontal forces to the vertical rope.

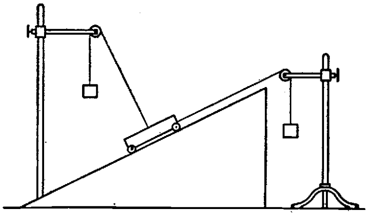
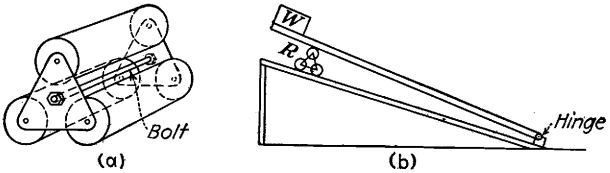


FIG. 6.—Forces parallel and perpendicular to the plane will support the car when the plane is removed.

M-19. Rolling Wedge. Three $\frac{3}{4}$-in. rollers about 6 in. long are mounted between pivot points near the corners of two triangular endpieces about 2 in. on a side held together by a long bolt (Fig. 7*a*). The wedge so made rolls down an inclined board; above this at a distance slightly less than the height of the wedge is hinged a second board, upon which a weight may be placed (Fig. 7*b*). The comparatively light wedge *R* will raise the hinged board and a heavy weight *W* as it rolls down the incline.



(a) (b)

FIG. 7.—The rolling wedge (*a*) lifts the weight as it descends the plane (*b*).

PARALLEL FORCES AND MOMENTS

M-20. Couple, a Free Vector. A strong permanent magnet is centered on a circular wooden or cork float. The magnet is set in the east-west direction, and upon release the float rotates about its center of gravity. The magnet is next placed on one end of a wooden strip laid across the float and counterbalanced by a brass bar at the other end. Upon release, the float again rotates about its center, in spite of the fact that the axis of the couple has been displaced. If small magnets are used, the whole experiment may be performed in a flat-bottomed glass dish and demonstrated by vertical projection.

With greater refinement, it may be shown that the magnitude of torque is the same whether the magnet is located on the axis or not by connecting a vertical torsion wire to the center of the float to determine the magnitude of torque required to hold the magnet perpendicular to the magnetic meridian.

M-21. Galileo Lever. A $\frac{1}{4}$-in. metal rod about 3 ft long is suspended by a cord tied to its center. A second identical rod, which has previously been cut in the ratio of 3:1 and then had the two parts threaded together or connected by a stiff bayonet joint, is suspended from the first by means of two equal lengths of string ending in rings, which can slip along the rods. First, the supporting strings are at the ends of each rod. Equilibrium obtains. Next the strings are slipped inward until each is vertical and at the center of one part of the lower rod. This rod is then separated into its parts, and equilibrium again obtains. The law of moments is demonstrated, as the forces applied to the upper rod are inversely proportional to their distances from the center.

M-22. Two pieces of 2-by-4, each 4 ft long (Fig. 8*a*), are fastened together by bolts passing through holes 6 in. from each end. The combination is hung from a cord attached to a screw eye in the middle of the upper piece, so as to be horizontal. The lower piece is cut through 1 ft from one end. The shorter piece is now supported from its center of gravity and can be turned about its supporting bolt without disturbing the equilibrium of the system; but if the longer part is turned, the equilibrium is upset.

M-23. Bridge and Truck Problem. Two kitchen platform scales with large dials reading to 10 or 20 lb are placed 5 or 6 ft apart. Across them is laid a light board, which rests on two triangular wooden fulcrums placed on the flat tops of the scales. The board may be stiffened by screwing to the under side a lengthwise strip, edge on. Heavy black lines 6 in. or 1 ft apart mark distances along the board. With the board in position, the pointers of the scales are adjusted to zero. A toy truck loaded with weights is placed directly over one scale. The class observes its weight. Then the dials of both scales are concealed, and the truck is drawn a quarter or a third of the distance along the bridge. The class is asked to compute the readings of the scales before the experimental values are revealed.

M-24. Action of a Torque. A large spool such as is used for wire has a string wrapped around it several times so that it will come off the bottom of the shank as the spool rolls along the table. Few students will guess correctly which way the spool will roll if the string is pulled parallel to the table. The spool may be made to roll in either direction by making the line of pull on the string cut the table to one side or the other of the line of contact of the spool with the table. If the two lines intersect, the spool slides. If a string is wound around each end of the shank, the tendency of the spool to turn about a vertical axis may be reduced (Fig. 8*b*).

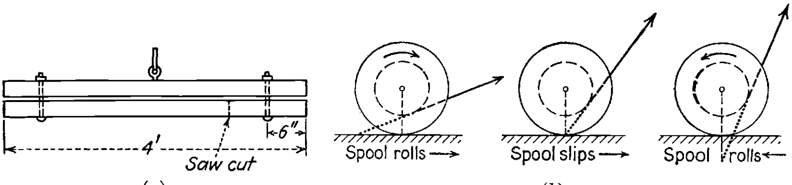


FIG. 8*a*.—Galileo lever. Turning the short piece of the lower board leaves the equilibrium undisturbed; turning the long piece destroys equilibrium.
FIG. 8*b*.—Angle of pull determines direction of motion.

This experiment may be made more effective if the spool rolls along a light horizontal plank suspended by four long cords. Then, if the radius of the shank is one-half that of the rim, a horizontal pull on the cord will not move the plank. If the radius of the shank is less than this, the plank will be moved in the direction of the pull; if it is more, the plank will move in a direction opposite to the pull. This experiment illustrates the apparently paradoxical case of a frictional force acting either in the direction opposite to the pull or in the *same* direction.

M-25. A bicycle is held upright with one pedal at its lowest point. The demonstrator pulls backward on a horizontal rope attached to this pedal. The bicycle moves *backward!*

M-26. Addition of Parallel Forces and Torques. A 4-ft board, 12 in. wide, is suspended horizontally from two spring balances hung from the ceiling or a supporting frame. Weights are laid on the board at various points, and the calculated and observed extensions of the balances are compared.

M-27. A meter stick is equipped with several light metal sleeves, which may be clamped in any position along its length and which are extended on either side by short strips of metal that serve as knife-edges. Wire stirrups ending in rings that slip over these knife-edges enable the meter stick to be supported at any point and also serve for attaching weights at various positions. The meter stick is supported from a spring balance hooked into the center stirrup, and weights are hung from the others until the stick balances; or the stick may be hung from two balances near its ends, and one or more weights may be hung from points between them. The observed and calculated values of the forces are compared. The weights of the sleeves and stirrups, unless very small, will have to be included. The sleeves should be symmetrical about their knife-edges. Loops of thread may replace the metal sleeves and stirrups.

A particularly instructive exercise is to support the meter stick off center and balance it with a single weight. The class is then asked to compute the weight of the meter stick from the positions of the known weight and the center of gravity of the stick.

M-28. Equilibrium of Moments. A drawing board is supported on a horizontal axis through its center, so that it can turn freely in a vertical plane. Weights are hung from threads looped over thumbtacks stuck into the board at various points and are adjusted until equilibrium is attained. The algebraic sum of the moments of all the forces is then shown to be zero. If the axis about which the board turns is hung in a wire stirrup from a spring balance, it can also be shown that the algebraic sum of all the forces (which in this case are all vertical) is also zero.

The experiment can be extended to include the case of nonparallel forces by running some of the strings over pulleys, but the adjustments and calculations are not so simple in this case.

M-29. Ladder Problem. A stick 12 to 15 in. long is hung by a thread from its upper end, and the lower end of the stick rests on a wooden float in a pan of water (Fig. 9). The stick may be inclined at any angle; but when equilibrium is attained, the thread is always vertical, since there can be no horizontal component of force at the float.

M-30. A short ladder, 6 or 7 ft long, is equipped with wheels or casters at its upper end so that as it leans against the wall, the reaction of the wall may be considered horizontal, tangential friction being negligible. The ladder is leaned against the wall at such an angle that it just does not slip. A cleat is firmly fastened to the floor about 6 in. from the foot of the ladder. A sheet of heavy galvanized iron will make the friction between the floor and the base of the ladder more dependable. The forces holding the ladder in equilibrium are discussed: (1) the ladder alone, (2) the ladder with the instructor standing on the first rung, (3) the ladder with the instructor standing on a higher rung so that the ladder will begin to slip. The demonstration consists in cautiously climbing the ladder until it slips. The cleat prevents a serious fall.

M-31. Center of Gravity. A flat irregularly shaped board is suspended from a point near one edge. The vertical direction through the point of support is indicated by a plumb line, and a line is drawn on the board to coincide with the vertical. This process is repeated with another point of support. The point of intersection of the two lines locates the center of gravity of the board. Lines similarly drawn from other points of support will pass through this intersection. If a nail is driven through the board at the intersection of these lines, it passes through the center of gravity; and if the ends of the nail are supported by two loops of string or by two horizontal rods, the board is in equilibrium in any position in a vertical plane.

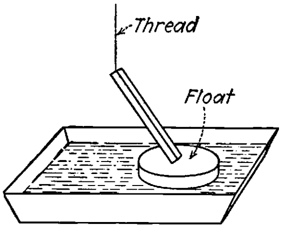


FIG. 9.—The float moves until the thread is vertical.

M-32. Equilibrium. Large models, a foot or more across, cut from plywood in various geometrical shapes, such as a triangle, semicircle, cycloid, or parabola, are suspended by means of a steel knitting needle thrust in turn through each of two or more holes several inches away from the center of gravity, which may be located as described in M-31. Another hole may be drilled at the experimentally determined center of gravity to show that each model is in neutral equilibrium when supported at that point. If desired, the experimentally located points may be compared with those determined by calculus.

M-33. A body is in stable equilibrium if its center of gravity is below the point of support, if a vertical line through the center of gravity passes within the area of support, or if any slight disturbance tends to raise the center of gravity. A familiar child's toy is a horse from the body of which extends a long bent rod carrying a lead weight at its end. The horse may be supported by resting its hind legs on a horizontal rod, a feat that would obviously be impossible without the bent rod and weight. The principle may easily be demonstrated with a block of wood and lead weight (Fig. 10) or with a wooden pencil and a jackknife.

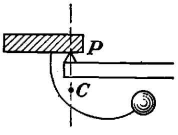


FIG. 10.—The heavy ball keeps the block in stable equilibrium.

M-34. Another demonstration of equilibrium consists of a truncated cone whose base is cut at an angle with the axis. The cone will rest in stable equilibrium upon this base; but if a short cylinder is fastened to the top by means of a peg in a hole, the center of gravity of the combination is no longer above the area of support, and the "tower" falls.

M-35. A thick wooden disk, 12 in. in diameter, is loaded near its periphery with a plug of lead. If it is placed on a gentle incline with the lead plug near the top, it may be made to roll uphill and will then come to rest with its center of gravity above the line of support in such a position that a slight displacement of the disk in either direction will raise its center of gravity, so that its equilibrium is stable.

The class may be alarmed to see the disk roll toward the edge of the lecture table, stop just short of the edge, and return. The starting point on the table and the corresponding point on the disk are of course found by trial and carefully marked in advance.

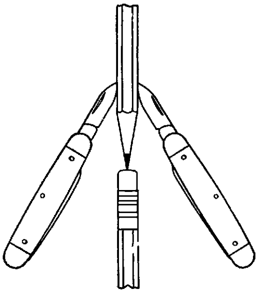


FIG. 11.—The center of gravity of the system is below the point of support.

M-36. A pencil, into which two jackknives have been stuck, may be balanced with its point resting on the end of another pencil (Fig. 11).

M-37. A double cone will appear to roll up an incline made of two gradually separating rails if the slope is such that the center of gravity of the body is slowly lowered as it rolls.

M-38. Two equal weights A and B (Fig. 12) are supported by vertical rods that slide in holes through the ends of a horizontal rod R and are held by setscrews. A third vertical rod passes through a hole in the center of the horizontal rod and can also be held by a setscrew. It carries a needle at its lower end, which rests on the head of a common pin driven into a vertical wooden support. The effects of raising or lowering the two weights and the point of support C on the equilibrium of the system are demonstrated. In the upper end of each rod is cut a notch, into which fits a tongue on a short section of the same rod. A pin is passed through the joint so that a hinge is formed. When the rods are slipped down until they hang from the short hinged section, the system is changed from stable to unstable equilibrium, although the center of gravity is far below the point of support.

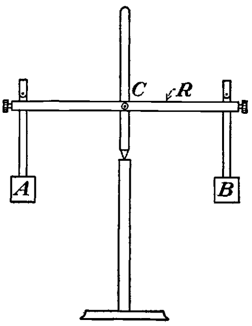


Fig. 12.—The center of gravity may be raised or lowered.

M-39. A cone turned from wood or metal serves well to illustrate stable, neutral, and unstable equilibrium.

M-40. Neutral Equilibrium. From a piece of 2- by 12-in. plank is cut an arc of the catenary $y = c \cosh x/c$, where c is 6 to 10 in. and the y-axis is perpendicular to the side of the board opposite the arc (Fig. 13). A similar plank is cut to form a polygon (*e.g.*, a rectangle) such that its center of gravity is at a distance c from one edge. The first board is set upright on the table and leveled if necessary so that the y-axis is vertical. The second board stands on the first so that its base is horizontal and its center of gravity lies in the y-axis (produced). Then if the polygon is rolled along the arc of the catenary it will remain in neutral equilibrium, since its center of gravity will move along a horizontal line. This can be shown by measurement with a meter stick, or by clearly marking the center of gravity and

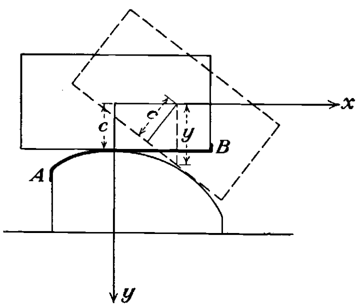
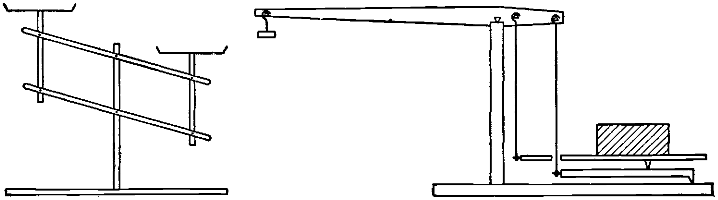


Fig. 13.—The rectangle is in neutral equilibrium on the catenary.

stretching a horizontal string at its height above the table, so that the mark will move along behind the string. As the curve becomes steeper, friction will no longer be sufficient to keep the board in place. To remove this difficulty, a piece of cotton tape may be tacked to the two boards in the position shown by the heavy line AB in the figure. Since its pull can only be tangent to the curve at the point of contact of the two boards, it can have no effect except to prevent slipping.

M-41. Balances. The principle and operation of balances may be demonstrated with as many types as are available. The steelyard consists of a graduated metal bar with a fulcrum near one end. This fulcrum is usually supported in the hand, the unknown weight is hung on the short arm, and a sliding weight is moved along the longer arm until equilibrium is attained. The arm is graduated to read directly the value of the unknown weight. The same design is employed in many other forms of balance, often with two or three sliding weights, each on a graduated arm, by means of which the range and accuracy of the balance may be increased.


(a) (b)
FIG. 14.—Platform balance (*a*), and weighbridge (*b*).

M-42. The equal-arm analytical balance may be demonstrated with the actual instrument or with the following model. A light wooden frame is constructed in the form of a T and is supported by a nail driven through the center of the crossarm. The stem of the T is tapered at the bottom to serve as a pointer. Weights are hung from the arm at equal distances from the support. A threaded rod carrying a heavy nut projects upward from the center of the crossarm. By varying the position of the nut, the height of the center of gravity of the system can be changed. Thus one may demonstrate both stable and unstable equilibrium as well as the effect on period and sensitivity of the relative position of center of gravity and point of support. The same effect may be demonstrated with an actual balance by replacing the weight attached to the pointer for adjusting the center of gravity by a much heavier weight so as to accentuate the effect.

Simple models made of wooden strips and brass pivots may be constructed to demonstrate the principles of various other types of balance, such as the drugstore or platform balance (Fig. 14*a*) and the so-called weighbridge (Fig. 14*b*), commonly used for weighing heavy loads.

MACHINES

M-43. Lever. The simplest machine is the lever. It may be demonstrated in simple form by a meter stick equipped with three movable knife-edges (M-27), one to serve as fulcrum, the other two to support weights. The three classes of lever can be demonstrated by the arrangements shown in Fig. 15. In each case, when equilibrium is established, the sum of the moments about any point is shown to be zero.

Such practical applications of the lever as a crowbar, wrecking bar, claw hammer, scissors, nutcracker, pointers on instrument dials (where the emphasis is on multiplication of motion or speed rather than force) may be exhibited.

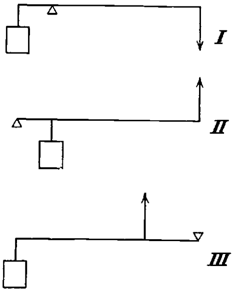


FIG. 15.—The three classes of lever.

The foot forms an interesting lever. How can the body be lifted by a muscle of which the upper end is attached to the body? If the foot rests on the floor, at a place three times as far from the ankle joint as the place of attachment of the tendon of Achilles, what is the force in that tendon if a man of weight 150 lb raises himself on the ball of one foot?

M-44. Windlass. A windlass is simply a lever arranged for continuous action. The fulcrum is the axis of rotation, and the lever arms are the radii of the crank and axle respectively. A model windlass may be demonstrated. The crank may be replaced by a pulley having a radius equal to that of the crank, in which case the machine is called a wheel and axle. Cords are wrapped around the wheel and the axle, and weights are hung on the cords to demonstrate the equality of moments. Mechanical advantage may be determined by finding the ratio of distances traversed by the two weights during a given motion of the wheel and axle.

M-45. Pulleys. As many types of pulley as are available may be demonstrated. Small models can be enlarged by shadow projection, but if possible full-sized blocks and tackles should be shown and heavy weights lifted by their aid (see M-17).

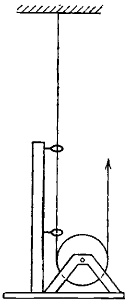


Fig. 16.—By standing on the platform and pulling upward on the rope, the operator "lifts himself by his bootstraps."

The differential pulley or chain hoist may be demonstrated, preferably with an actual machine.

M-46. The way in which the mechanical advantage of a pulley system depends upon the number of ropes supporting the weight may be shown by means of a boatswain's chair. This is simply a 2-ft board, on which the demonstrator sits, held in a large loop on the end of a rope. The rope passes over a pulley fixed to the ceiling, and the demonstrator asks one of the lighter members of the class to raise him. After he has failed, the demonstrator raises himself with one hand.

A variation is shown in Fig. 16. One end of the rope is fixed to the ceiling, while the other end passes through a pulley attached to a platform on which the demonstrator stands. By pulling on the free end of the rope, he can "raise himself by his bootstraps." It is well to have the rope pass through a sleeve on a post several feet high rigidly fastened to the platform, to prevent the platform from tipping.

M-47. Let the student who has had experience in calculating mechanical advantages of ordinary tackles try his ingenuity on the arrangement shown in Fig. 17, sometimes called "fool's tackle." If necessary, set the arrangement up with pulleys and weights.

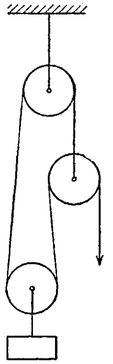


Fig. 17.—"Fool's tackle."

FRICTION

M-48. Tribometer. An inclined plane of wood, covered with metal on one face, is hinged at the bottom so that its angle can be varied and so that either face can be made the upper one. Smooth blocks of wood or metal can be laid on the plane, and the angle varied until they will just slip down without acceleration after they have been started. The coefficient of sliding friction is then equal to the tangent of the angle of slope. In the same way, the coefficient of starting friction can be determined and shown to be larger than that of sliding friction.

The difference between starting and sliding friction may be shown with a piece of soft cloth (*e.g.*, a bath towel) laid over a long smooth rod or a piece of straight glass tubing 2 in. in diameter and 5 or 6 ft long. One end of the towel may be perhaps 6 in. lower than the other, without the towel slipping off the glass. However, if the tube is inclined enough so the towel slides along, then it will also run across the tube and fall off on one side. While moving lengthwise of the tube, it easily moves crosswise.

M-49. To show that sliding friction is independent of the area of contact between the two materials, a wooden block a foot long is cut in the form of a prism whose cross section is an irregular polygon. The angle of slope of the inclined plane at which the block will slide uniformly is the same whatever face it may be resting on. The same fact may be shown by pulling the block across the top of the lecture table with a spring balance. As long as the speed is constant, the reading of the balance is the same regardless of the area of contact.

M-50. Dependence of Friction on Pressure. Hold a meter stick horizontally on the index fingers of the two hands, one finger at 10 cm, the other at, say, 70 cm. Ask the class on which side the stick will fall as the two fingers are brought slowly together. The two fingers always meet at the middle of the bar, with the bar still poised above them. The same thing is true even though the stick is supported on substances as dissimilar as rubber and steel.

M-51. Angle of Friction. A pencil is held by the point with the rubber eraser resting against the table; starting in the vertical position, the pencil is pushed longitudinally and is inclined until it slips. The angle between pencil and vertical at which slipping occurs is the angle of friction (M-48), and its tangent is equal to the coefficient of friction of rubber against the material of the table. A slight twisting motion is sufficient to overcome starting friction.

M-52. Snubbing. The application of friction to the method of snubbing may be demonstrated. More reproducible results are secured if instead of a wooden post and ordinary rope one uses a polished laboratory support rod about an inch in diameter with tightly twisted cord like fishline. The rod is mounted horizontally, and a light spring balance is attached to one end of the cord. Weights are hung on the other end, and the weight required to produce a given reading on the balance is plotted against the number of turns around the snubbing post. The frictional force rises rapidly with the number of turns around the rod.

M-53. Automobile Skidding. The effect on the course of a skidding automobile of locking front or back wheels is demonstrated with a small model. Either pair of wheels may be locked by thrusting a pin through them into holes in the body. The car is released upon a steep incline made of wallboard. It either straightens out or skids further according as front or back wheels are locked.[1]

M-54. Friction Pendulum. Suspend a ball on a thread from a loop of thread 4 to 6 in. long, hung over a round horizontal wooden bar that can turn in bearings. As the bar turns slowly, the pendulum starts oscillating and builds up a large amplitude. The alternate drag and release of the thread by the turning bar may be compared with the intermittent friction between a violin bow and string.

M-55. Fluid Friction—Viscosity. A cylindrical vessel containing a liquid (water or oil) is mounted on a vertical axis so that it can be rotated. Inside the vessel and coaxial with it is a second cylinder supported by a torsion fiber carrying a small mirror. When the vessel is rotated, friction in the liquid causes a drag on the inner cylinder, which is indicated by the motion of a spot of light reflected from the mirror. This principle is used in one type of viscosimeter.

M-56. A flat sheet of metal supported by two long threads is hung in a vertical position in a glass projection cell containing water or glycerin. A metal disk is mounted so that it is parallel with the sheet and partly dips into the liquid in the cell. When the disk is rotated, the sheet is displaced in the direction of motion of the disk in the liquid, as may be shown by projection.

M-57. Effect of Temperature on Viscosity. If a cylinder containing castor oil (M-55) is placed within a water bath on a

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[1] JONES, A. T., *Am. Phys. Teacher*, **5**, 187, 1937.

horizontal turntable rotated by a friction-drive motor, the effect of temperature on viscosity may be shown. As the water bath is heated from 5 to 40°C, the viscosity of the castor oil falls off in a ratio of 15:1.

M-58. Fall of Pressure in Pipe. A number of equal small holes are bored in a long horizontal ¾-in. pipe, and the pressure drop in the pipe due to friction as water runs through it is indicated by the decreasing heights to which the water rises from the holes toward the exit end of the pipe. A piece of shallow gutter pipe running to the sink will serve to catch both the water flowing through the pipe and that coming from the holes. Illuminating gas flowing through a ⅜-in. pipe will show the same effect by the heights of flames at a series of holes along its length (L-31).

M-59. A more ambitious scheme is to have a pipe connected to the water mains running up one side of the lecture room, across the top, and down the other side, and finally emptying into a sink. Four pressure gauges, one at the top and one at the bottom of each side, indicate the pressure. The difference between static and kinetic pressure is clear at once when the water faucet at the sink is turned on.

M-60. Egg Torsion Pendulum. Two eggs, one raw and the other hard-boiled, are supported in wire baskets by similar torsion wires. Each is set oscillating as a torsion pendulum; the hard-boiled egg continues for some time, but the motion of the raw egg is quickly damped by internal friction. Similarly a hard-boiled egg may be spun on a plate much more readily than a raw egg. Once spinning, however, if the raw egg is stopped by touching it momentarily with the finger, it may resume rotation when released.

M-61. Viscosity in Gases. Frictional drag in gases may be shown by fastening two vertical parallel sheets of metal to one arm of a balance, with a disk arranged to rotate between them. The displacement of the parallel sheets in the direction of motion of the part of the disk between them is made evident by a disturbance of the equilibrium of the balance. This and the earlier experiment (M-56) may be interpreted in terms of the transfer of momentum from layer to layer of the fluid.

M-62. A metal disk is hung horizontally by a three-point support from a single vertical thread. A short distance below it is located a parallel disk driven by a motor. The upper disk, which carries a prominent index, may be displaced or even set into rotation by the frictional drag of the air.

ELASTICITY

M-63. Hooke's Law and Young's Modulus. A length of wire is hung from the ceiling (avoiding kinks), and a weight pan is attached to the bottom. Near the bottom of the wire is clamped a small collar with a wedge-shaped top, on which rests one end of a horizontal brass bar 10 cm long (Fig. 18*a*). The other end of the bar is supported by a knife-edge or by two pointed legs and carries a small vertical mirror from which a spot of light is reflected to a screen, thus forming an optical lever by means of which the motion of the end of the wire is made visible. Weights are hung on the weight pan, and the elongation of the wire is shown to be proportional to the force applied. That the elongation is also proportional to the length of the wire may be shown by using wires of different lengths. From the constants of the optical lever, it is possible to compute the actual elongation under a given load and thus to compute Young's modulus for any wire. The optical lever and its support may be easily transferred from one wire to another if their lower ends are all near together. For shorter wires, this necessitates hanging them from some support lower than the ceiling.

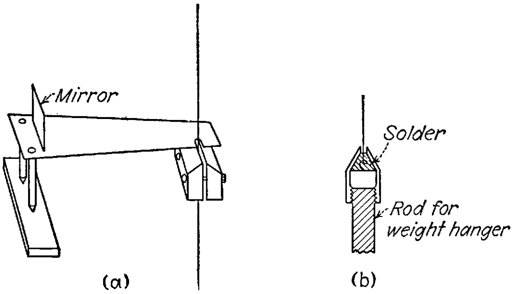

FIG. 18.—Young's modulus. Optical lever for showing elongation (*a*) and method of holding end of wire (*b*).

The chief difficulty in this experiment is to hold the ends of the wires satisfactorily. Clamps will either slip or pinch the wire so that it breaks easily. A satisfactory support is shown in Fig. 18*b*.

A piece of brass rod has a cone turned on one end. The other end is drilled and tapped, and the cone is drilled out, with a hole through the apex somewhat larger than the wire to be used. The end of the wire is pushed through the hole, and a knot is tied in it. The knot is drawn part way into the cone, and the latter is filled with solder. A threaded rod is screwed into the tapped end to serve at the top for holding the wire or at the bottom for holding the weight pan.

If desired, the weight may be increased until the yield point of the material is reached. With one sample of copper wire (No. 20 d.c.c.) this was about 7000 g, but it varies considerably. The wire suddenly stretches very much more than before. As weights are added, the great increase of length continues; but the modulus of elasticity changes as well, so that the force needed to stretch the wire continually increases. A piece of copper wire may be stretched as much as 20 per cent or more of its original length before it breaks. It is of interest to show with a micrometer caliper that the diameter of the stretched wire is markedly less than that of the unstretched wire.

Experiments like the preceding for the verification of Hooke's law can also be performed with spiral springs of various materials and stiffnesses and with rubber bands or tubes.

M-64. Bending, Shearing, Torsion. A rectangular block of rubber is very convenient for demonstrating various kinds of strain and the stresses that produce them. If a sufficiently large block is not available, rubber erasers with the ends cut off square may be passed around the class or demonstrated by opaque projection. It is particularly interesting to show the change in the shape of cross section under bending and torsional stresses.

M-65. A pile of paper, cards or glass plates, or a thick book with flexible binding may be used to demonstrate the sliding of elementary planes over one another (due to shearing stress). A similar model to show torsion may be made of a number of slotted weights set on top of one another with a rod passing through their central holes. Another rod is placed in the slots, where it must fit loosely. When the top weight is twisted, all the others will also be turned through progressively smaller angles.

A very effective way of showing bending and other strains is by means of models made of transparent plastic and projected with polarized light (L-133).

M-66. The bending of rods and bars of various materials and cross sections may be demonstrated. The specimen may be supported from knife-edges at various separations, or it may be clamped at one or both ends. Various loads may be hung from the free end or the middle, and the amount of bending indicated by some form of optical lever or measured directly with a micrometer screw mounted vertically above the point of loading. Another method is to use shadow projection. At the point of loading, a small vertical piece of card or metal is attached to the beam or rod. A second similar piece is held just above the first and in the same plane by means of an independent support. The shadow of the two is cast on the screen, and the height of the second is adjusted until only a fine line of light appears. From the widening of this line when the specimen is loaded, the bending will be apparent.

M-67. Deformation under Stress. A pattern (*e.g.*, a circle, an inscribed square, and a set of radial lines) is painted on a sheet of rubber cut from an old inner tube. Opposite edges of the sheet are shellacked and held between two strips of wood tightly bolted together. Changes of shape of the pattern when the two sticks are pulled apart show the effect of deformation under stress. Nonuniform changes may be demonstrated with a similar sheet in which only one edge is held clamped while forces are applied to the corners or elsewhere.

M-68. Impact. Elastic and inelastic impact may be demonstrated with two heavy steel balls supported by bifilar suspensions so that they just touch one another. If the balls are of the same mass, when one is drawn aside through a measured arc and allowed to swing and hit the second, the latter will be driven out through nearly the same arc while the first will be brought to rest. If the balls are of different masses, bearing a simple ratio to one another, the laws of momentum transfer (M-124 and M-127) may still be verified approximately. Inelastic impact may be illustrated by sticking a small bit of modeling clay or soft wax to one of the balls at the point of contact so that the two balls stick together after impact.

M-69. The coefficient of restitution may be determined approximately by dropping balls on a horizontal plate. Glass marbles may be dropped on a piece of glass, or steel balls on a heavy smooth steel plate. The plates should be rigidly mounted on an unbending support. Indexes attached to a vertical meter stick are used to indicate the initial height and the height of rebound. The square root of the ratio of height of rebound to height of fall is the coefficient of restitution. The apparatus should be brilliantly illuminated to increase visibility, or shadow projection should be used.

VELOCITY

M-70. Measurement of High Velocity. The shaft of a synchronous motor is extended at each end, and to each of these extensions is attached a disk of polar coordinate paper about a foot in diameter. The axial distance between the disks may be made 1 ft. Bullets are fired through the disks parallel to the axis, both when the motor is at rest and when it is rotating at a known speed. The change in angular position of the bullet hole in the second disk is made obvious to the class by passing straight rods through the two pairs of holes. By laying one disk upon the other in such a position that a nail may pass through the holes made when the motor was at rest, the angle through which the disks turned while the bullet was in flight between them may be measured, and hence the time of flight through a measured distance. If a synchronous motor is not available, any type of motor may be used and its speed of rotation measured with a tachometer.

M-71. Observation of Low Velocities. Project as for microscopic objects (A-50) the movement of the end of the minute hand of a clock. It will probably be necessary to extend the minute hand with a bit of wire for convenience in projection.

M-72. Velocity Magnitudes. Make a diagram showing typical velocities expressed in centimeters per second:

	Cm per sec
Geologic movement	3×10^{-7}
Glaciers	2×10^{-5}
Growth in plants	10^{-4}
Point of hour hand of watch	3×10^{-4}
Point of minute hand of watch	5×10^{-3}
Snail	15×10^{-2}
Point of second hand of watch	3×10^{-1}
Blood	7
Snowflake	20
Light wind	5×10^{2}

	Cm per sec
Sprinter	10^3
Hurricane	5×10^3
Earthquake wave	3×10^4
Moon around earth	10^5
Earth around sun	3×10^6
Halley's comet near sun	4×10^7
Cathode rays	3×10^9
Light	3×10^{10}

M-73. Vector Addition of Displacements. A bead is pulled vertically along a wire rod fixed in a frame that may move horizontally. The string lifting the bead passes over a pulley on the frame and is made fast to the fixed coordinate system (Fig. 19). The vertical lift of the bead, therefore, equals the horizontal movement of the frame, and the line $y = x$ is plotted.

If now the bead is made a movable pulley by passing the string around the bead and tying it to the frame, the vertical movement is halved, and the bead traces the line $y = x/2$. A small frame approximately 6 in. square is suitable for shadow projection.

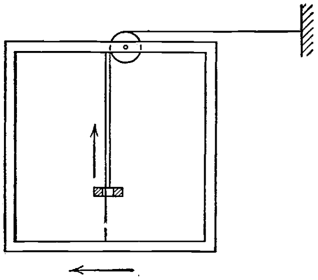
FIG. 19.—Vector addition of displacements.

M-74. A ball is tied at the center of a piece of twine 6 ft long, by means of which it may be pulled in either direction through a horizontal glass tube 2 ft long. By holding one end of the string against the table and raising the tube, the ball may be made to follow the resultant of a horizontal and a vertical motion.

M-75. Vector Addition of Velocities. A small rug or piece of canvas, across which runs a slow-moving mechanical toy, is drawn at uniform speed along the lecture table. The resultant motion of the toy relative to the table may be indicated by a chalk line connecting the start with the finish.

UNIFORM ACCELERATION

M-76. Self-timing Inclined Plane. A wide shallow rounded groove is cut lengthwise in the face of a 2-in. plank, 6 ft long and 8 in. wide. The groove is painted black. A heavy iron ball 3 or 4 in. in diameter, rubbed well with chalk, is rolled down the plane so as to oscillate from side to side of the groove. The ball leaves a wavy trace. The increasing length of the waves shows the increasing distance passed over in the constant time of oscillation, from which the constant linear acceleration may be determined. The *difference* between successive half wave lengths is constant.

M-77. Timed-interval Inclined Plane. Two so-called "highly polished rounds" of steel, ½ in. by 12 ft, clamped closely together on a plank, form an excellent groove for rolling a 1-in. steel ball. Small lights flashed at regular intervals are attached to the plank at such points that a flash occurs each time the ball passes one of the lights. Neon or flashlight lamps are suitable, and the timing may be done by a pendulum with a mercury contact at the lowest point of its swing. The acceleration may be calculated from the increased distance passed over in successive equal time intervals.

M-78. Inclined Wire. A small car suspended from overhead wheels slides down an inclined steel piano wire stretched very taut by turnbuckles. The wire should be about a millimeter in diameter, and the car as light as possible. This setup may be permanently attached close to one wall of the lecture room, with large numbers painted thereon to show the time-distance relation. A metronome designates the proper equal time intervals.

M-79. Guinea-and-feather Tube. That the acceleration of free fall is independent of mass is readily shown by quickly inverting a long evacuated glass tube of large diameter containing a bit of lead and a feather. It is best to demonstrate the fall of the two bodies both before and after exhausting the tube, to show the contrast.

M-80. Rate of Fall Independent of Mass. An iron ball and a wooden ball of the same size into which is driven an iron plug are suspended at the top of the lecture room from the poles of an electromagnet. A bit of paper between each pole and ball insures quick release when the magnet circuit is broken. The balls fall together and simultaneously strike a tin pan placed on the floor beneath the magnet.

M-81. Suspend from the lecture-room ceiling two simple pendulums of equal length, one with a large cork bob, the other with a bob of lead. Pull them to one side 5 ft or more, and release simultaneously. The bobs fall back to their equilibrium position together.

M-82. Velocity Acquired by Ball on Inclined Plane. The velocity acquired by a ball rolling down an incline may be found, as Galileo determined it, by measuring the distance traversed by the ball in one unit of time along a horizontal extension of the track. This extension is slightly inclined so that a ball rolls along it without acceleration. The ball is released on the incline at such a height that its passage to the horizontal track coincides with a click of the metronome. A movable block placed on the track is then adjusted until the next click of the metronome coincides with the impact of the ball on the block.

M-83. A more spectacular experiment (similar in purpose to M-82) consists in arranging a seconds pendulum (approximately 1 m long) so that it is held aside by an electromagnet and released by the passage of the rolling ball from the incline to the horizontal track. The position of the pendulum is adjusted so that as it swings across the track it strikes the rolling ball laterally.

M-84. Freely Falling Bodies. A series of lead or iron balls, $\frac{1}{2}$ to 1 in. in diameter, is threaded on a fishline with their positions fixed by knots so that the intervals between them increase in arithmetical progression; 6, 10, 14, 18, . . . , in. are suitable intervals. The string is attached to the ceiling so that the lowest ball is 2 in. above a tin pan placed on the floor. The balls start falling simultaneously when the upper end of the string is released. They strike the pan with a constant frequency of impact (in this case, approximately 10 per sec). The more balls the more effective the experiment. By passing the string through a hole in the ceiling or by hanging it in a stair well, the advantage of additional height is gained.

M-85. Freely Falling Slab. A synchronous or properly timed motor is mounted with its axis vertical. On the upper end of its shaft is mounted a small ink bottle with a side tube (a bit of $\frac{3}{4}$-in. brass rod with a $\frac{5}{8}$-in. hole drilled in it serves for the bottle, and an 8-32 screw pierced with as small a hole as possible serves for the side tube). A cylindrical guard in the plane of the rotating nozzle protects the clothing of the experimenter. A slab of wood, $\frac{1}{2}$ by 2 by 18 in., has a strip of paper tacked to it and is dropped between the guard and the rotating nozzle. The ink squirted from the bottle cuts the falling slab in a series of nearly horizontal parallel lines separated by ever increasing vertical distances. If the speed of the motor is known, the acceleration of free fall may be determined from measurement on these lines.[1]

M-86. Pendulum-timed Free Fall. A rectangular iron block (2 by 5 by 10 cm) is hung by two steel wires so as to swing like a pendulum (Fig. 20). A thread attached to the back of the block by a screw eye passes around horizontal rods A and B and over a hook at C to a 1-in. metal ball. The position of the hook C is such that the ball is tangent to the vertical plane through the front face of the block in its rest position. The pendulum is drawn aside, and the length of the thread adjusted until the ball hangs at a point whose height above the rest position of the center of the block is $\pi^2 l/8$, where l is the length of the pendulum. The weight of the ball and friction on the rods A and B are sufficient to hold the pendulum aside. The face of the block is covered with slips of paper and carbon paper held by rubber bands. The thread is burned near C so as to release ball and pendulum simultaneously. The pendulum strikes the ball laterally, and the point of impact is clearly marked by the carbon paper. Thus h, the distance the ball falls, is determined, and t, the time of fall, is equal to a quarter period of the pendulum. The acceleration of gravity is readily calculated from the equation $h = \frac{1}{2}gt^2$.

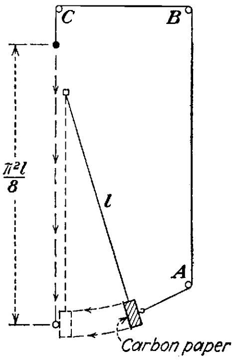

FIG. 20.—Pendulum-timed free fall.

M-87. Free-fall Apparatus. Another arrangement for finding the acceleration of gravity is shown in Fig. 21. Two magnets are connected in series; the upper retains a steel ball, while the lower retains an arm that carries a stylus in contact with a revolving drum. When current is interrupted, the ball starts to fall, and the stylus arm is released simultaneously, thereby making a mark on the revolving drum. The ball falls upon the platform and causes the stylus to make a second mark on the drum. The time of fall may be directly determined from the positions of these two marks and the speed of rotation of the drum.

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[1] For reduction of data, see Millikan, Roller, and Watson, "Mechanics, Molecular Physics, Heat and Sound," p. 25, Ginn and Company, Boston. 1937.

M-88. Galileo's Inclined Plane. A stiff circle, 4 or 5 ft in diameter, is constructed from metal rod or strap. Upon tightly drawn wire chords, radiating every 15° from the top of the circle, a set of glass or metal beads (Fig. 22) slides freely. If the beads are released simultaneously, their locus is a circle of increasing diameter that finally coincides with the stiff frame. Thus it is shown that the time of fall along any chord is independent of its slope.

M-89. In another method, the circle and chords are laid out upon a large piece of plywood, stiffened by backing strips to insure that the sheet remains plane. A series of circles tangent at the top is drawn upon the board, and the chords are delineated by means of No. 14 copper wire drawn tightly between holes. The plane is inclined with a slope of 1 in 10 or 20, so that $\frac{1}{4}$-in. balls will roll down the wire guides. Initially, the balls are retained within a small ring, coincident with the first circle, which is quickly but cautiously raised so as to set all the balls in motion at once. Stops at the ends of the various chords should give off sufficient sound so that the simultaneous arrival of the balls is made audible by a single click. The board may be reversed so that the common point of the chords is at the bottom. If the

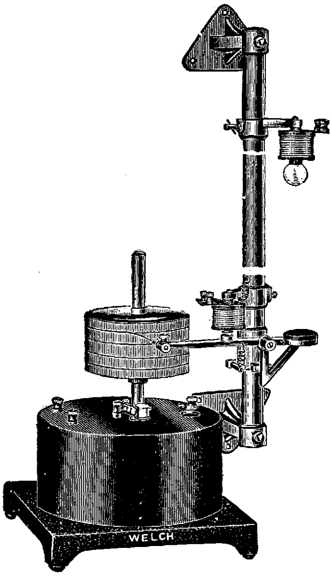

Fig. 21.—The hinged arm drops when the ball is released and rises again when the ball strikes it. Its motion is recorded by the stylus on the revolving drum.

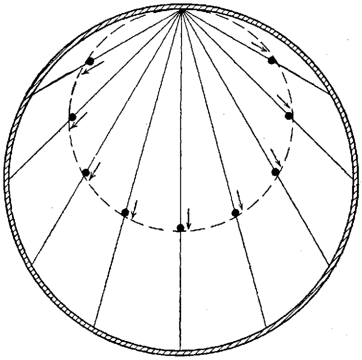

Fig. 22.—Balls are released simultaneously on all the chords. As they fall their locus is always a circle.

balls are all started from any one of the circles, they reach the common point simultaneously.

MOTION IN TWO DIMENSIONS

M-90. Water-stream Parabola. A glass or metal nozzle, connected by rubber tubing to a supply of water (either a tank of variable height or the water mains from which the flow can be controlled by a valve), is attached to a stick so that initially the jet of water is parallel to the length of the stick. Suspended by threads along the stick at equal intervals, measured from the orifice, are balls or rings hanging at lengths respectively proportional to the numbers 1, 4, 9, 16, . . . n^2. The stream may be adjusted by a pinchcock so that it passes through the rings or just beneath the balls however the angle of elevation of the jet may be changed, since the balls always lie on a parabola. For large classes, shadow projection may be used to show the parabolic path of the stream.

M-91. Rate of Fall Independent of Horizontal Motion. Two simple types of apparatus are available for dropping one ball vertically at the moment when another is projected horizontally. The first consists of a horizontal shaft, which when released by a trigger is forced by a compression spring into an endwise movement. From one end of the shaft, a ball is pushed off as the shaft moves into one bearing. The other end of the shaft strikes another ball as it comes out from the other bearing.

The second type consists of a board at one corner of which are placed the two balls. One is held over a hole by a retaining spring strip; the other is ready for projection. A flexible metallic strip with a projecting screw is drawn back and released. As it strikes the ball to be projected, the projecting screw strikes the strip retaining the other ball and releases it. Both balls may be heard to strike the floor simultaneously.

M-92. Falling Target. A $\frac{1}{2}$-in. brass tube 2 ft long supported in a laboratory stand serves for the gun (Fig. 23). The target, not more than 3 in. square, is a sheet of iron suspended from a doorbell magnet. A bit of paper inserted between target and pole pieces insures prompt release of the target when the current is interrupted. The gun is bore-sighted at the target, and the aim is demonstrated to the class by passing a beam of light from a pocket flashlight through the tube to the target. The bullet consists of a cylindrical brass plug or a steel ball that fits the tube smoothly. A No. 40 copper wire in series with the electromagnet is stretched on an insulating frame across the center of the muzzle of the gun. The projectile is inserted in the gun and blown toward the target, either by the lungs or by compressed air. On emerging from the tube, it cuts the wire and breaks the magnet circuit, releasing the target. The bullet hits the falling target. The experiment is equally successful with different angles of elevation of the gun and different muzzle velocities. A laboratory apron makes a good backstop.

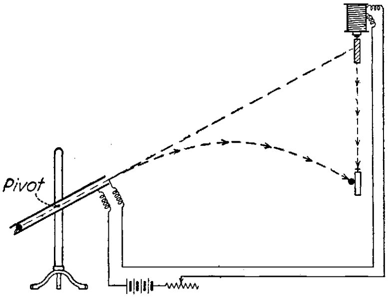

FIG. 23.—The target is released when the bullet leaves the gun. The bullet always hits the target.

There are numerous variations and refinements of this popular experiment. In one the projectile is a ball that acquires its initial velocity by rolling down a steep incline; it leaves the incline on a line directed at the target, which is a second ball.[1] In another, the gun is simply an inclined groove along which a steel ball is propelled by a ruler; the target is a golf ball. In a third, the projectile is a pointed dart, blown from a tube or shot from a crossbow, and the target is a cat's fur held to the magnet by a piece of iron. In a fourth, the magnet circuit is opened by a carefully adjusted switch or by a pressure-controlled contact that operates at the instant when the projectile passes a small hole near the end of the gun tube. In a fifth, the air gun and the magnet are attached to a long 2-by-4 so that, regardless of its angle of elevation, the gun is always aimed at the target.

M-93. Brachistochrone. Two $\frac{1}{4}$-in. polished-steel rounds, bent to form a cycloidal track and laid side by side in contact, may be attached to the wall or a wooden frame. An excellent groove is thus formed in which to roll $\frac{1}{2}$-in. steel balls. The cycloid should be of such an extent that even near its ends the balls roll without slipping. Two balls started simultaneously at points on opposite sides of the cycloid always strike in the center,

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[1] SCHILLING and EICKHOFF, *School Science and Math.*, January, 1934.

a "dead heat" no matter what the handicap. The motion of a single ball rolling to and fro is isochronous, as in the cycloidal pendulum (M-94).

Two straight rounds, clamped together, may be pivoted at the lowest point on the cycloid so as to form an incline of variable pitch. Steel balls may thus be raced from any point on the cycloid by the two paths. The ball rolling on the cycloidal path always wins.

M-94. Cycloidal Pendulum. A circular piece of plywood 2 ft in diameter is rolled without slipping along the frame above the chalk rail at the lower edge of the blackboard. It is pushed along by a peg through its center, and the cycloid is described by a piece of chalk placed in a hole near the periphery. Two other pieces of plywood shaped in the form of a cycloid may be shown to fit this curve and then inverted to form a cusp, between the sides of which a simple pendulum may be made to oscillate with large amplitude (Fig. 24). Direct comparison with the small-angle swings of a simple pendulum of the same length but without the cycloid attachment shows that the cycloidal pendulum is isochronous, regardless of amplitude. If both pendulums are swung through large arcs, the cycloidal pendulum gains on the simple.

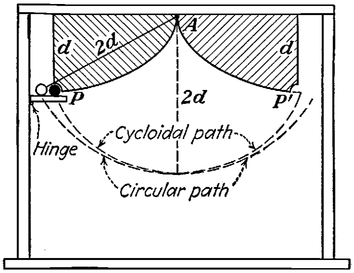

FIG. 24.—Comparison between simple and cycloidal pendulums.

M-95. Range of a Projectile. A toy popgun, spring pistol, or simple crossbow is adjusted to shoot a projectile at an elevation of 45° as far as conveniently possible within the confines of the lecture room. The elevation is then shifted to 30° and 60° and the new range compared with the calculated value. A chalk box placed at the computed range will catch the projectile.

M-96. Parabolic Path of Projectile. A spring gun mounted at one corner of an inclined sheet of plywood is used to fire a steel ball, which may be caught in a box placed at the correct distance. The ball will trace its parabolic path if it rolls over a fresh piece of carbon paper covering a sheet of white paper tacked to the board. The ball should be heavy and the angle of the plane not too large.

RELATIVE MOTION

M-97. Acceleration Relative. A 1.5-in. glass tube 1 ft long with its lower end plugged with a metal disk is suspended close to the ceiling by a thread. A metal ball is also suspended by the thread so as to hang within the tube. The thread is cut above the point *A* (Fig. 25), and both tube and ball fall freely. The tube is caught in a well-padded box or in the hand, and the ball will be seen and heard to strike the metal end *after* the tube is caught.

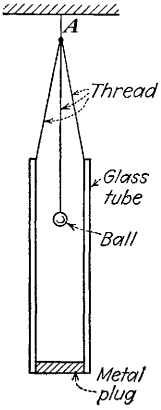

FIG. 25.—When the thread is cut above *A*, ball and tube fall with the same acceleration.

M-98. Freely Falling Candle. A steadily burning candle is placed inside a lamp chimney closed at the bottom. If the candle and chimney are dropped 2 to 3 m, the candle goes out, not because of air currents blowing the flame but because of the absence of convection currents around the accelerated candle.

M-99. Relativity Car. A spring gun is mounted vertically upon a car that rolls along a horizontal track. The trigger of the gun is tripped by a projection between the rails of the track as the car reaches a certain point (Fig. 26). The gun projects a steel ball some 3 ft upward between two rings vertically in line with the gun and attached to a mast fixed to the car. The muzzle of the gun is enlarged with a funnel to insure catching the ball on its return. If the motion of the car is uniform, the gun catches the ball regardless of the speed of the car. The distinction between the fixed and moving axes is emphasized if the car with mast removed is obscured by a screen during the flight of the ball.

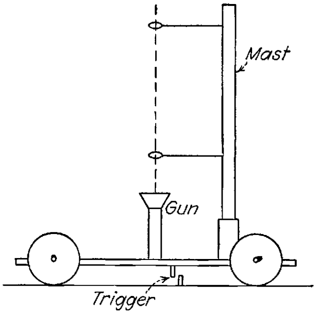

FIG. 26.—When the car is at rest or in uniform motion, a ball shot vertically from the gun falls back into the muzzle.

MASS OR INERTIA

M-100. Inertial Reaction. Short pieces of the same stout string are attached to screw eyes on opposite sides of a large mass, *e.g.*, a 16-lb shot. The ends of these strings are tied in loops with a bowline knot[1] so that a $\frac{3}{4}$-in. laboratory rod may pass through them. The weight is suspended by one loop from a hook on the wall. A rod is inserted through the lower loop, and the class is asked whether it wishes the string broken above or below the ball. A steady downward pressure on the rod plus the weight of the ball will snap the string above. Raising the rod a few inches and then giving it a quick downward flip will break the lower string. By a sufficiently quick jerk it is possible to break two strings below the weight while it is supported by only a single string. A padded box is used to catch the weight, or the weight itself may be wrapped with rags.

M-101. Breaking a Rope by Inertia. A heavy iron ball is hung from a hook by a light cord. One end of a piece of ordinary clothesline is tied to a screw eye in the ball; the other end is attached to the head of a hammer. A full-arm swing of the hammer away from the ball takes up the slack in the rope at the moment when the hammer acquires its highest speed. The clothesline may thus be snapped in two without disturbing the heavy ball appreciably.

M-102. Vibrograph. A mass of several kilograms rests upon a glass plate that is supported by two steel needles (diameter about 0.5 mm). These needles are free to roll on a second glass plate placed on the table top. The upper part of the system remains at rest relative to the floor when the table is set in vibration by a hammer blow. The resulting motion of the needles is made evident by reflecting light from a mirror cemented to one of them. Fundamentally, the system is a vibrograph, accelerometer, or seismograph. A rotating mirror (S-76) may be used to show a time trace of the motion.

M-103. Weight vs. Inertial Reaction. A heavy weight is lifted slowly from the floor with a light cord. If, however, the cord is suddenly pulled, it breaks without lifting the weight. Similarly, a 10-lb weight is suspended from a spring balance; by suddenly raising or lowering it, the balance reading may be doubled or halved. The experiment demonstrates in simple fashion the reason for one's sensations in an accelerated elevator.

M-104. Inertia Tricks. A tumbler of water stands on a sheet of paper resting near the edge of the lecture table. The free end

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[1] See "Boy Scout Manual," Boy Scouts of America, New York.

of the paper is held in the right hand at about the level of the table top. The paper between the right hand and the tumbler is struck a sharp downward blow with the left hand. The paper comes out from under the tumbler, and the latter is hardly moved.

Snap a card out from under a coin. If the card rests on a tumbler, the coin falls therein; or the card and coin may be balanced on the finger tip. The coin remains after the card departs. A white silk "tablecloth" may be suddenly jerked from under tools, beakers of colored water, etc., set on the table.

Even the initiated who know the coin-and-card trick may fail in an attempt to remove a slip of paper from beneath a *tall* metal cylinder without upsetting it. The essential element of success is a *sudden* impulse of short duration, more sudden than ordinary hand jerks can give. Place a tall cylinder (4 by $\frac{1}{4}$ in.) on top of a long slip of paper, allowing the end of the slip to project beyond the table edge. Strike the projecting paper a sharp downward blow with the wetted finger tips as the hand descends from a full-arm swing.

Two similar mercury jugs are placed side by side on the table. One contains mercury or lead shot; the other is empty. A student is asked to seize and lift one jug with each hand quickly at the instant a signal is given. The result is surprising to all. Naturally, the student must not know beforehand which jug is heavier.

An apparent departure from Newton's laws of motion may emphasize to students how intuitively they accept those laws from their everyday experience. If a ball is at rest, they expect it to remain at rest; if it is rolling across the table, they expect it to continue in a straight line with only slight retardation by friction. It therefore astonishes an unsuspecting class to see a ball suddenly start rolling or another swerve aside as it rolls across the table. A strong electromagnet planted beneath the lecture table at a spot inconspicuously marked so that the instructor alone knows its position may be controlled with a foot switch so that a steel ball will perform unexpected motions in its field. A piece of plate glass carefully leveled on the table reduces frictional and gravitational influences on the ball.

M-105. Inertia Cart. A horizontal bar is arranged to rotate freely about P, the vertical axis of a post mounted on a cart or car

(Fig. 27). At the extremities of this bar are pins over which may be slipped cylinders C_1 and C_2 of equal volume but different mass. As the car is accelerated, the greater mass lags behind, but the reverse happens on stopping.

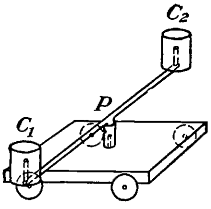

Fig. 27.—Inertia cart.

M-106. Inertia Balance. The concept of inertia, as distinct from that of weight, may be clarified by means of an inertia balance[1] in which the force due to gravity plays no part. The balance consists of a horizontal metal bar 20 cm long supported from a heavy stand by two 25-cm spring-steel strips (Fig. 28). The body whose inertia is to be determined is set on the horizontal carriage, and the balance is made to vibrate in a horizontal plane. The period of the balance is measured with a stop watch. The inertia M is then given in terms of the period T by the equation $M = aT^2 - b$, where a and b are positive constants. Thus the inertias of several bodies, *e.g.*, those comprising a set of weights, may be compared without recourse to the forces of gravity upon them. The constants of the balance may be determined by assigning a value of inertia to some body that is to be taken as a standard.

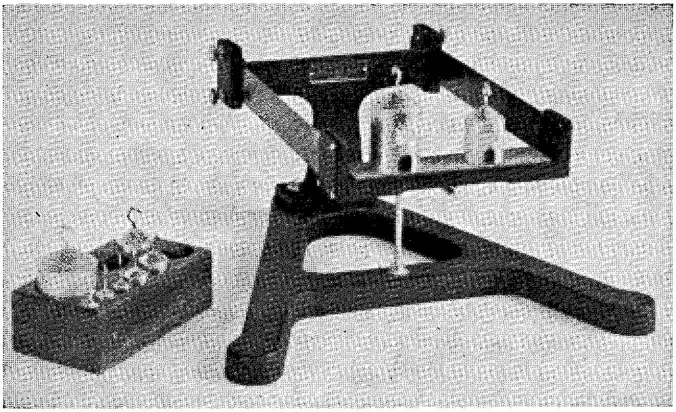

Fig. 28.—Inertia balance.

M-107. Static vs. Dynamic Forces. Drop a 50- to 100-g cylinder on a flat surface of modeling clay, well-worked putty, or soft wax. Move the cylinder to another spot, and cautiously lay

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[1] Schriever, W., *Am. Phys. Teacher*, **5**, 202, 1937.

weights on top of it until a depression of the same depth results. This may necessitate adding many times the weight of the cylinder itself, thus showing the large force brought into play by sudden stopping of the falling cylinder.

NEWTON'S SECOND LAW

M-108. Acceleration on a Horizontal Plane. A car is accelerated along a horizontal track by a string passing over a pulley and tied to a weight. The accelerating weight and the mass of the car may be varied. The acting force may be made of longer duration if the cord passes over a pulley high above the lecture table (Fig. 29). The acceleration may be measured by marking the table with chalk to indicate the position of the car at several successive clicks of a metronome; or the acceleration may be made immediately visible by means of a liquid accelerometer (M-288) carried on the car.

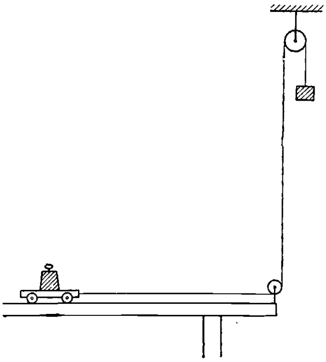

FIG. 29.—The car is accelerated by the descending weight.

M-109. Negative Acceleration Due to Friction. A block of wood is drawn over the table top by a spring balance and the force of friction observed. Then it is placed on the board of a rigid swing that comes to rest just at the level of the table top (Fig. 30). The swing is drawn aside a known distance, released, and allowed to collide with the table. The block is thus shot from the swing with a known initial velocity and comes to rest after skidding across the table. The acceleration is computed and the force equation checked. If desired, the energy aspect of this experiment may be emphasized, since a measurable amount of potential energy is wasted in frictional work.

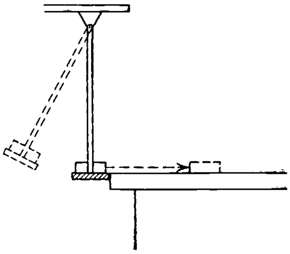

FIG. 30.—Negative acceleration due to friction.

M-110. Atwood's Machine. At the top of a large scale (divisions about 10 cm apart), which reaches from floor to ceiling, is mounted a light, carefully balanced pulley. Two equal combinations of weights are suspended from the ends of a string passing over the pulley. About one twenty-fifth of the weight on one side is removed and placed on the other side. The heavier weight is raised to the ceiling. A metronome is set beating seconds. After instructing successive rows of the class to observe the position of the rising weight at the end of successive seconds, the instructor counts "zero" as he releases the weight. The first row observes the position of the weight when the instructor counts "one," the second row when he counts "two," etc. The observations of each row are quickly averaged, and the acceleration ($g/25$) is computed from the observations and compared with the value obtained from $F = ma$. Allowance for friction may be made by adding a small weight to the descending side so that the system moves in the desired direction without acceleration prior to shifting weight from one side to the other.

M-111. Tension in Atwood's Machine. When the weights are at rest, each cord is under a tension equal to the weight it sustains; but when the weights are released, the tensions in the two sides become equal, since $m_1(g + a) = m_2(g - a)$ for the rising and falling weights respectively. This fact may be shown by hanging the weights from large-dial spring balances on the two sides.

M-112. Double Atwood's-machine Problem. Over a light balanced pulley hangs a cord, with masses of 1 and 2 kg, respectively, at its ends. This pulley is supported from one end of another cord, which passes in turn over a fixed pulley with a 3-kg mass tied to its other end (Fig. 31). The first pulley is prevented from turning by a pin between wheel and yoke. Enough mass is added to the 3-kg mass to balance the suspended pulley, and the class is asked to predict the motion of the 3-kg mass when the pin is removed and the two smaller masses are free to accelerate. The arrangement may be hung in front of the blackboard, and the original positions of the masses marked thereon.[1]

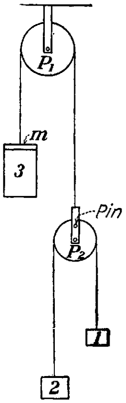

FIG. 31.—Double Atwood's machine.

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[1] For the analysis of a similar problem, see W. E. Byerly, "Generalized Coordinates," p. 19, Ginn and Company, Boston, 1916.

M-113. Climbing-monkey Problem. "A monkey hangs on a rope that passes over a frictionless pulley to a coconut that weighs just as much as the monkey. How can he get the coconut?" This provocative experiment can be shown with varying degrees of elaborateness. The system consists of two equal masses balanced from the ends of a cord hung over a light fixed pulley. One of the masses is equipped with a mechanism for winding up the cord. As the cord is wound up, *both* masses rise at the same rate.

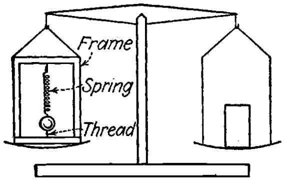

FIG. 32.—Disturbance of equilibrium by acceleration.

In the simplest arrangement, a spring tape measure, or a spring chain such as is used for holding eyeglasses, plays the role of the monkey. In another form, a brass rod, 50 to 75 cm long and weighted at its lower end, is drawn up into a brass tube, into which it fits loosely, by means of an old shade-roller spring. The spring is previously stretched by pushing the rod down and locking it with a trigger. Upon release of the rod, the weight is accelerated upward. A yet more complicated arrangement uses a small electric motor with shaft geared down to a spindle upon which the string may be wound up. This should preferably be a toy motor run with flashlight batteries, in which case the "monkey" is completely isolated, or it may be a more powerful type with the current leads as light as possible and so hung as to impede the "monkey's" progress as little as possible. If an electric motor is used, the direction of motion may be reversed.

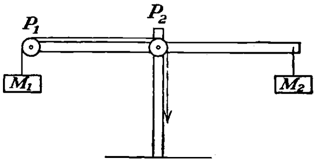

FIG. 33.—If M_1 is at rest or in uniform motion, the balance is in equilibrium.

M-114. Acceleration on a Balance. A wooden frame is placed upon one table of a platform balance or in one pan of a large beam balance and brought to equilibrium (Fig. 32). Within the frame, a large ball, 3 to 5 kg, is suspended on a spring and pulled, stretching the spring farther, by a string that is tied to the bottom of the frame. The string is burned and the motion of the balance observed.

M-115. Reaction Balance. A pulley P_1 is fastened to one end of a stiff bar, which is supported as the beam of an equal-arm balance, with a second pulley P_2 centered at the fulcrum (Fig. 33). A mass M_1 is hung from a string passing over the two pulleys, and an equal mass M_2 is hung from the other end of the balance beam. When the end of the string is held fast or is moved with uniform velocity, the balance is in equilibrium. But when the end of the string is pulled so that M_1 moves with acceleration either up or down, the equilibrium is destroyed.

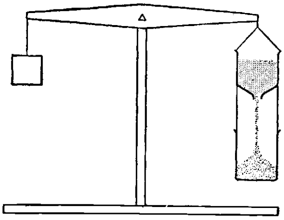

FIG. 34.—Galileo's balance.

M-116. Galileo's Water Balance. Galileo suspended from one arm of a beam balance two buckets, one above the other (Fig. 34). The upper contained water, the lower was empty. When the water was allowed to run out of the upper bucket into the lower, Galileo expected to see that arm of the balance drop on account of the impact. Actually it rose slightly when the water began to flow, but as soon as the water reached the lower bucket, equilibrium was reestablished. The experiment may also be done with sand.

M-117. Gas-pressure Analogy. Pressure caused by change of momentum of numerous bombarding particles may be shown by pouring lead shot steadily upon the apex of a cone mounted on a float in a vessel of water (Fig. 35). The resulting transfer of momentum causes a depression of the float. There is a close analogy between this experiment and the kinetic theory of gas pressure, as both the number of the particles striking the cone per second and their velocity may be varied. A similar (gaseous) reaction may be shown by causing an idle electric fan to rotate in the wind stream of another fan.

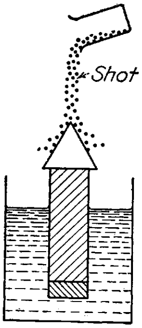

FIG. 35.—Gas-pressure analogy.

ACTION AND REACTION

M-118. Reaction Carts. Two light planks, 6 to 8 ft long, reinforced with cantilevers to prevent excessive bending, are equipped with rubber-tired wheels. Two students stand on the carts holding a rope between them. If the students weigh the same, then as both pull the rope, the carts meet midway between their original positions. The carts are again separated, and the rope is made fast to one cart while the student on the other pulls the rope. The carts meet at the same spot as before.

A man standing on a cart at rest may start to run along it. The cart moves in the opposite direction. Conversely a man running along the floor may stop on the cart. The cart moves, of course, in the direction of his original motion. With the two carts tied together, two men may start from the ends toward the center. Finally, a man running at *constant* speed may, in the course of crossing the room, run along a cart without moving it, since there is no transfer of momentum if his speed is constant.

M-119. Rocket Car. A heavy medicine ball may be tossed from one man to the other as they stand on two carts, with resulting transfer of momentum. The two men may throw a baseball back and forth. One man supplied with a dozen baseballs may throw them at some protected spot on the wall.

M-120. Reaction Swings. Experiments similar to those performed with the reaction carts may be done with swings, consisting of two planks each suspended from the ceiling by means of four equal ropes so that they hang about 6 in. above the floor. Or a seat may be placed on one swing upon which the demonstrator sits. He pushes on a heavy iron weight suspended from the ceiling. In case the demonstrator throws baseballs from one swing, it may be possible by proper timing to build up a considerable amplitude.

M-121. Reaction Cars. Two small cars connected with a rubber band are placed on a track a meter apart (Fig. 36). They are released simultaneously, and the stretched band pulls them together with accelerations inversely proportional to their masses. The masses are adjusted to simple ratios. The point at which the cars will meet may be predicted; for since the same force acts on each at every instant, the distances traversed are inversely proportional to their masses.

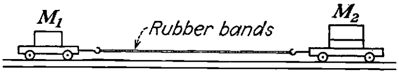

FIG. 36.—Reaction cars.

In place of a stretched elastic band, a string may join the two cars. It passes over two fixed pulleys in the middle of the track and then to a movable pulley attached to a descending weight.

M-122. Cannon Car. A small brass cannon mounted on one car fires a lead bullet into a block of wood mounted on another (Fig. 37). The powder is set off by a gas flame on the end of a long tube. The class should be protected by a sheet of heavy plate or safety glass. If the masses of the cars are equal, they will recede from one another at equal speeds. If the two cars are tied together with wire, they will not depart from their original positions despite the firing of the cannon, showing the equality of action and reaction.

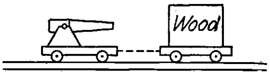

Fig. 37.—Action and reaction.

M-123. Reaction Track. About 4 ft of straight toy track is mounted on a light board, which in turn is placed on steel rollers. A toy locomotive of either the spring or electric type and a car or two, properly weighted, run along the track. The track moves in the opposite direction, thus showing the resultant reaction. Extra rollers should be in line to take care of the movement of the board, which may be beveled on the end to insure its taking the rollers smoothly.

Conservation of angular momentum (M-184) may be shown in similar fashion by mounting a circular track on a bicycle wheel free to revolve about a vertical axis. By changing the loading of the train, it is possible to control the velocities imparted to both train and track relative to a fixed point.

IMPACT AND MOMENTUM

M-124. Velocity of Bullet by Ballistic Pendulum. A gun or revolver carefully mounted fires a bullet into a ballistic pendulum consisting of a block of wood about a foot long turned to fit tightly into a length of iron pipe 4 or 5 in. in diameter (Fig. 38). This pipe is suspended horizontally from the ceiling by four parallel wires, whose lengths may be adjusted by violin pegs. The horizontal deflection of the block is recorded by the position of a light rod, which is pushed by the pendulum into a tube supported on a stand. It is advisable to push the rod almost to its final position before firing the gun, in order to avoid spurious results due to friction or to momentum imparted to the rod. The equation for the velocity v of the bullet is $v = \frac{M + m}{m}\sqrt{\frac{g}{l}}d$, where M is the mass of the pendulum, m the mass of the bullet, l

the length of the supporting wires, and *d* the displacement of the pendulum.

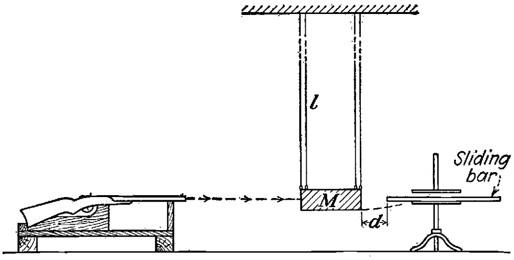

Fig. 38.—Ballistic pendulum.

M-125. Momentum Pendulum. A heavy pendulum bob is suspended from a light wooden framework mounted on a flat board (Fig. 39). On the bottom of the board are two metal strips that rest on steel rollers; these in turn rest across steel rails. When the pendulum oscillates, changing the direction of its momentum, the periodic movement of the frame shows the reaction upon the support.

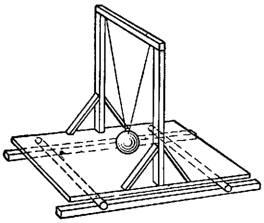

Fig. 39.—Momentum pendulum.

M-126. Momentum Cars. The same effect as with the momentum pendulum (M-125) may be shown by two cars resting on a level track. They are joined by a light elastic band stretched between uprights. A motor mounted on one is geared to a wheel that carries an eccentric arm reaching to the other. With the motor running, the cars alternately separate and approach. The system as a whole is at rest, as is shown by a light rider hung on the elastic band to indicate the point of no motion. The relative masses of the cars may be changed by moving a weight from one to the other, thereby changing the position of the point at rest (center of gravity of the system).

M-127. Impact Pendulums. Two steel or ivory balls are hung by bifilar suspensions. The suspending threads should make a considerable angle with one another, so that the balls swing accurately in a plane. The balls should just be in contact at their equilibrium positions. Velocities at the time of contact are, to a first approximation, proportional to horizontal displacements from rest positions. Cardboard uprights set on the table help to make these distances evident. Several different cases of impact may be shown. In elastic impact with equal masses, one being at rest and the other in motion or both being in motion, the velocities are interchanged. A bit of wax on the balls makes the collision inelastic with a common final velocity. In every case, whether elastic or inelastic, momentum is conserved. Particularly fascinating is the special case of elastic impact with the masses in the ratio of 3:1 and initially colliding with equal but opposite velocities.[1] The heavier ball is stopped while the lighter ball flies back with twice its former speed. After the second collision, the balls return to their original state, and the cycle repeats itself.

GRAVITATION

The demonstration of universal gravitational attraction requires apparatus too sensitive for the rough-and-ready work of the lecture room. However, permanent setups may be mounted on the walls of the laboratory where the students may operate them at their leisure.

M-128. Gravitational Torsion Balance—Cavendish Experiment. A long torsion pendulum made of a quartz fiber or a fine tungsten wire supports a mirror and light dumbbell suspension bearing two small lead balls (Fig. 40). The whole is rigidly fixed high on the wall with an optical system of high magnifying power to show the deflections of the mirror.[2] Sliding on parallel bars are two large lead balls, actuated from below by means of cords, so that their attractions on the suspended balls may be reversed. The period of the torsion pendulum is too long to make a determination of the gravitational constant feasible during a lecture, but the motion of the suspended system under gravitational force may be readily shown by projecting a spot of light from the mirror upon the screen. The constants of the instrument and the theory of the experiment may be posted near by so that any interested student may actually work out the first figure of the gravitational constant by observing the deflections of the mirror. In demonstrating the principle of the apparatus, a large-scale model of the torsion pendulum may be shown and analyzed. (M-167).

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[1] Lemon, H. B., *Am. Phys. Teacher*, **3**, 36, 1935.

[2] Lemon, H. B., *Am. Phys. Teacher*, **2**, 10, 1934.

M-129. Variation of Pendulum Period with Field of Force. Suspend an iron ball by a long wire over a large electromagnet, concealed in a box. Start the pendulum swinging, and then energize the magnet. By placing a rheostat in series with the magnet, the period of the pendulum may be varied over a wide range. This experiment simulates the variation of weight with height and may be used to illustrate the pendulum method of measuring the mass of the earth. The observed variations in

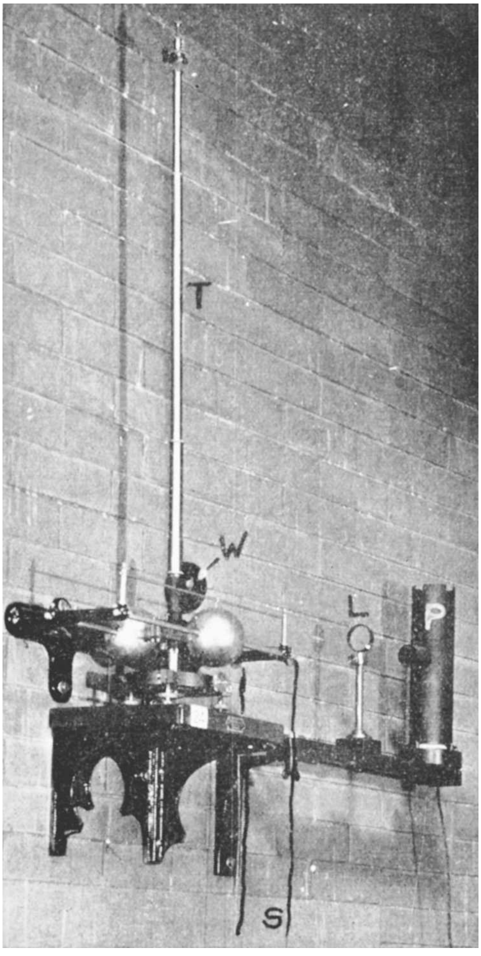

FIG. 40.—Cavendish balance.

period caused by the electromagnet are, of course, far greater than the variations produced by changes in g due to change in altitude.

M-130. Motion under Inverse Square Law of Attraction. Motion of one electrically charged ball in the electrostatic field of another and of one magnet in the magnetic field of another are described in A-62 and A-63. However, the motion is only a crude approximation to planetary motion because of the relatively large and inconstant component of gravitation force present.

M-131. Elliptic Motion with Conservation of Angular Momentum. A steel ball rolled in a glass funnel (M-144) or in a rigid paper cone with axis vertical will, under certain conditions, describe an elliptic motion with conservation of angular momentum and (except for frictional losses) conservation of energy. Differences in kinetic energy at "perihelion" and "aphelion" are accounted for by differences of gravitational potential energy.

WORK AND ENERGY

Most of the experiments already described may be presented from the point of view of energy. This is particularly true of the following experiments listed: 19, 37, 43–47, 82, 109, 110, 112, 161–166, 185, 216, 232–237, 239, 279.

M-132. Galileo's Pendulum Board. Hang a simple pendulum from a peg at the top of the blackboard. Pull the pendulum aside so that it makes an angle of 30 or 40° with the vertical. Draw a horizontal line at the level of the bob. Release the pendulum, and show that it rises again to this line on the opposite side. Now intercept the string supporting the bob in the middle of its swing by a pencil held firmly against the board. The bob, in spite of the shorter radius of swing, rises to the height from which it started. This is true also of the swing in the reverse direction. If the pencil is placed low enough, the bob makes a complete loop.

If desired, the apparatus can be mounted on a board about a meter square, with a dowel inserted in turn into a series of holes arranged vertically below the point of support.

M-133. Pile Driver. Dissipation of potential energy in friction and useful work may be shown by driving a nail into a block of soft wood parallel to the grain, by means of an improvised pile driver, consisting of a 1-lb steel block, 1.5 or 2 in. in diameter. The block is guided by a tube some 2 ft long into which it fits very loosely. It is allowed to fall vertically upon the head of the nail. A series of 10 or 12 blows will drive the nail into the block. A measurable advance is shown after each blow. An enlarged shadow projection of the nail makes the motion clearly visible to the class. A rough plot of friction against length of nail in the wood clearly emphasizes how work is the integral of force times distance.

M-134. Brake Horsepower Test. A standard test of the power delivered by a motor can be carried out in a few minutes. A pulley at least 3 in. in diameter is attached to a $\frac{1}{2}$-hp motor. A strip of leather belt or brake-lining material about a foot long is looped around the pulley like a belt and is attached to the hooks of two spring balances, one of which is made fast to the ring of a turnbuckle so that the tension on the belt may be adjusted. The delivered horsepower is computed from the measured values of pulley diameter d, revolutions per second n, and difference in tension on the balances $T_1 - T_2$, by the equation

$$\text{HP} = \frac{\pi n d(T_1 - T_2)}{550},$$

where d is in feet and $T_1 - T_2$ in pounds.

M-135. Power Developed by a Person. A brake similar to that described in the preceding experiment may be fitted over a wooden disk with a metal rim about 2 ft in diameter. A slate disk or even an old grindstone may be substituted. The disk is mounted on substantial bearings and fitted with a crank. The tension in the brake is adjusted so that not over three turns per second are required to absorb the power output of professor or student. A student may, of course, compute his own "horsepower" by measuring the time required to run up two or three flights of stairs and by combining this observation with that of his weight and of the height he climbs.

M-136. Power Bicycle. The rear sprocket of an old bicycle is firmly bolted to the end of a wooden axle, 2 ft long and 2 in. in diameter, mounted in suitable bearings. The axle is driven by the chain from the front sprocket of the bicycle. A stout rope made fast to this axle passes over a pulley in the ceiling and is attached to a heavy weight. By measuring the distance the weight rises in a given time, the power developed by the bicycle rider may be determined.

ROTATION

For many of the experiments that follow, a rotating table with good bearings is essential. This may be a stool on which the instructor may stand or a chair suspended from a swivel in the ceiling. In some laboratories, such a stool has been constructed from the front-wheel bearing and wheel of an automobile.

Various types of motor-driven variable-speed rotator are available. A small motor, such as is used in electric fans, whose axle is fitted with a rubber stopper, is convenient for spinning gyroscopes and other apparatus. A small hand drill is sufficient for producing rotation in many instances.

M-137. Centrifugal Force vs. Centripetal Force. A wooden ball is rotated by hand in a horizontal plane at the end of a thread 2 ft long. The demonstrator holds in his other hand a long, sharp knife with which to cut the thread. While swinging the ball overhead, he discusses with the class what will happen to the path of the ball if he cuts the thread at some predetermined point. Many students have the feeling that centrifugal force is a force that will continue to act after the constraining centripetal force vanishes. The demonstrator stands in such a position that the ball would fly into the class if centrifugal force continued to act after centripetal force ceased. The ball, of course, continues tangentially after the thread is cut and may be made to strike some protected spot.

M-138. Measurement of Centripetal Force. The magnitude of centripetal force required to constrain a body to a circular path may be shown with very simple apparatus, consisting of a 20-cm length of glass or metal tube through which passes a 1-m length of fishline. To one end of the line is attached a 1-kg weight; to the other end, a wooden ball or other object weighing 50 to 100 g. The tube is held in a vertical position and moved by hand in such a way as to set the light ball rotating in a horizontal circular path. The weight is lifted from the floor or table by centrifugal force. By timing the number of revolutions per minute with the ball swinging around a circle of known radius (marked on the string), it may be shown within 1 or 2 per cent that the number of revolutions per minute is inversely proportional to the square root of the radius of rotation. The period of revolution is $t = 2\pi\sqrt{ml/Mg}$, where m and M are the small and large masses respectively, and

l is the length of the cord from the axis to the center of gravity of the revolving mass. Commercial apparatus in which the force is measured by the extension of a spring or by lifting an axial weight may also be shown (M-177).

M-139. High-speed Chains. A loop of brass safety chain is made to fit rather loosely on a 6-in. wooden disk fastened to the shaft of a high-speed motor with controlling resistance. The chain is gradually given a high rotary speed and then cautiously forced off the disk with a screw driver. The rotating chain maintains its circular shape as it rolls across the lecture table. If a piece of 2-by-4 or other obstacle is placed in its path, the chain will bounce like a rigid hoop. If a longer piece of chain, looped over the disk and an idler pulley held by hand, is set going and released, it will roll across the table, preserving its original oval shape. The necessary centrifugal force and the normal component of the tension providing it are each inversely proportional to the radius of curvature of the chain at any point. It is well to have a rubber mat under the drive wheel so that the chain may acquire its speed of translation quickly without scratching the table. Caution is necessary, as these chains sometimes burst or fly off in unexpected directions.

M-140. Paper Saw. A 6-in. disk of typewriter paper spun at high speed possesses surprising rigidity. It will sound metallic when struck and will cut through other paper. Larger disks of Bristol board or heavier paper may be used to cut through wood.

M-141. Rotating Candle Flame. A lighted candle protected from air currents by a lamp chimney is placed on the rotating table at some distance from the axis. The flame will be found to point toward the center of the table. A flame placed 1 ft from the axis and rotated at 1 rps shows the effect nicely.

M-142. Paraboloid of Revolution. A cylindrical glass jar is accurately centered on a rotating table. Colored water is placed in it, and the cylinder is rotated. After viscous forces have brought the water to a steady state of revolution, the surface becomes a paraboloid of revolution (Fig. 41). If the jar is just half full at the beginning, the vertex of the paraboloid will reach the bottom of the jar just as the liquid reaches its upper edge. A short lighted candle on the end of a stick may be lowered to the bottom of the jar to show the absence of liquid at the center.

Clear water covered with colored castor oil so as to fill completely a cylindrical jar closed at the top will, on rotation, show a paraboloid of oil surrounded with water. Any two immiscible liquids of different densities may be used.

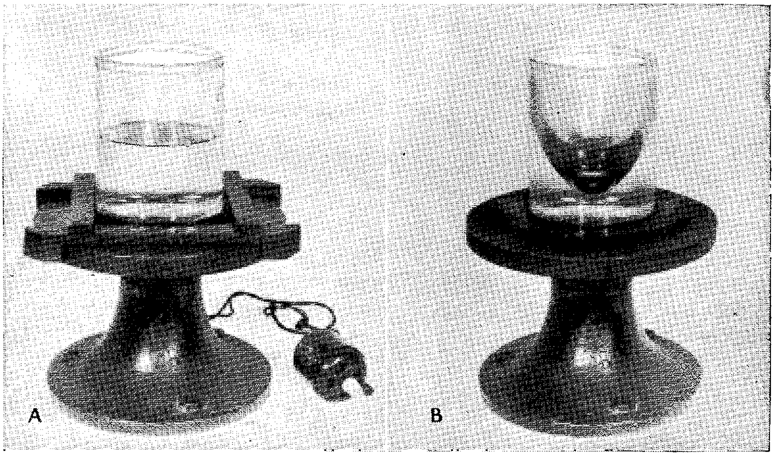

FIG. 41.—*A*. Battery jar with water at rest. *B*, same in rapid rotation showing paraboloid of revolution.

M-143. Parabolic Mercury Mirror. A circular pool of mercury may be rotated to make a parabolic mirror (L-25). The focal length of the mirror is $g/8\pi^2n^2$, where n is the speed of rotation in revolutions per second. A 6-v automobile headlight bulb is placed above the center of the mirror, and the mercury is rotated at constant speed for several minutes. The height of the bulb is then adjusted until the reflected light just fills a circle of the same diameter as the pool of mercury. The measured height of the bulb above the vertex of the mirror and the computed focal length should agree closely.

M-144. Motion on a Banked Curve. A steel ball rolled down a short incline enters tangentially into a large glass funnel with axis vertical. The ball spirals downward but rapidly increases its speed of revolution until it reaches an equilibrium level where it revolves in a horizontal plane. As its speed diminishes owing to friction, it slowly spirals to the bottom of the funnel. This simple experiment may be used to show why curved roadways and railroad tracks should be banked.

M-145. A variation of this experiment consists in throwing a steel ball or shooting it with a blowgun tangentially into a megaphone or other large conical tube with axis horizontal. The ball is shot toward the narrow end of the cone, but contrary to the expectations of the class, it emerges from the wide end. The initial direction of the ball should make a small angle with a generatrix of the cone. It is evident that the ball, as it spirals into the narrow end of the megaphone, encounters a component of centrifugal force directed toward the wide end. It is virtually impossible to make the ball emerge from the mouthpiece unless it is initially directed very close to a generatrix or unless the speed of the ball is *low*. High speed defeats itself.

M-146. Centrifugal Force in Pendulum Support Wire. A heavy iron ball is supported by a wire whose breaking strength is less than twice the weight of the ball. The ball is pulled aside by a force at right angles to the wire until the wire makes an angle of 60° with the vertical, whereupon it is released. The tension in the wire increases as the ball descends in its circular path, until the wire breaks. For if the ball is released and the supporting wire does *not* break, the tension in the wire becomes twice the weight of the ball when it passes through the rest position.

M-147. Flattening of the Earth. The conventional demonstration of the circular spring hoop that flattens on rotation may be varied by rotating a large ball of clay and glycerin suspended by cloth tape from a rotator. The ball will flatten perceptibly upon slow rotation, and as the speed increases, it will burst. A solid sponge-rubber ball suspended from the shaft of a motor by a short length of piano wire and spun at high speed may be deformed until the ratio of its equatorial to its polar diameter is greater than 2:1. When rotation ceases, the ball resumes its spherical shape.

M-148. Box-office Value of Centrifugal Force. A celluloid doll clothed with a full skirt, the hem of which is loaded with shot or beads, is set into rotation by a hand drill. A slight spin shows why front seats in a theater are preferred.

M-149. Stroboscopic Demonstration of Dynamic Distortion. A wheel with eight spokes is cut from sponge rubber $\frac{1}{4}$ in. thick. One of the spokes is severed near the rim of the wheel. The wheel is rotated at high speed on the shaft of a motor and is viewed first under steady and then under intermittent light

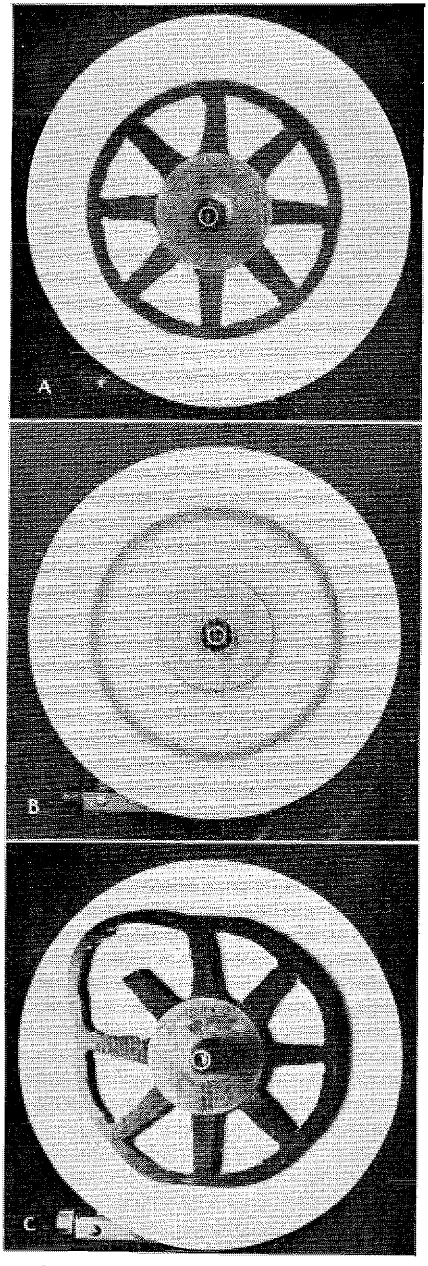

FIG. 42.—Rubber wheel with one spoke cut. *A*, at rest; *B*, rotating rapidly and illuminated with a steady light; *C*, same as *B* but under stroboscopic illumination, showing distortion.

(S-49). The motion of the wheel is "stopped" with the stroboscopic light, and the large distortion due to centrifugal force that failed to appear under ordinary light is made evident (Fig. 42).[1]

M-150. Speed of Rotation by Manometer. A U-shaped manometer containing water or mercury is mounted with one of its arms coincident with the axis of a rotating table. The necessary centripetal force on the liquid in the horizontal tube is introduced by the difference in level in the two arms, a quantity that is easily observed by the fall of liquid in the axial arm of the U. The depression in level of the liquid in the axial arm of the U is $\Delta h = \omega^2 r^2/4g$, where r is the length of the base of the U, and ω is the angular speed in radians per second. It will be observed that Δh is independent of the density of the liquid filling the manometer (M-289).

M-151. Simple Centrifugal Air Pump. Three sheet-metal disks are set mutually perpendicular and rotated about a vertical axis along the intersection of two of them. The resulting motion of the air may be shown, first, by holding a lighted match over the pole of rotation and observing how the flame is drawn downward; and second, by placing a flat paper ring, whose inside diameter is a little greater than that of the disks, around the equatorial disk, where it will be supported by air pressure. (This latter behavior is, incidentally, a good illustration of Bernoulli's principle.)

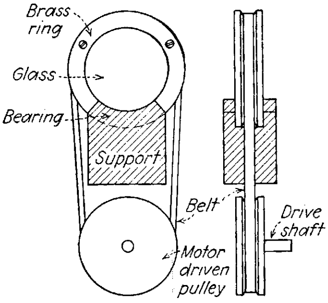

Fig. 43.—Projection centrifuge.

M-152. Projection Centrifuge. The clarification of a liquid by centrifuging may be shown by a projection-cell centrifuge consisting of two parallel, circular glass plates cemented in a brass ring (Fig. 43).[2] The plates are 1 cm apart and 7 cm in diameter, with the space between them nearly filled with the liquid to be separated. *Lotio alba* is satisfactory. The cell is held between guides and supported on a brass bearing of about 90°. There is a groove on the outside of the brass ring to take the

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[1] Hitchcock, R. C., *Elec. Jour.*, **32**, 529, 1935. Acknowledgment is made to *The Electrical Journal* for permission to reproduce the illustrations in Fig. 42.

[2] Erikson, H. A., *Am. Phys. Teacher*, **6**, 39, 1938.

drive belt from a small motor-driven pulley. As the cell rotates, an air bubble appears first at the center; then a halo begins to surround the bubble as the liquid clears; finally the clear liquid is surrounded by a ring of dark precipitate.

M-153. Model Centrifuge. A glass tube 1 in. in diameter and 12 in. long is bent as shown (Fig. 44). It is partly filled with water, and two balls, one of aluminum, the other of cork, are sealed inside it. The cork floats in one arm while the aluminum rests at the bottom. But when the tube is whirled, the aluminum is thrown to the outer end of one arm, while the cork floats on the *under* side of the water in the other arm, thus illustrating the separation according to densities in a centrifuge. Mercury instead of aluminum and a cork ball in each arm will show the same effect with better dynamic balance.

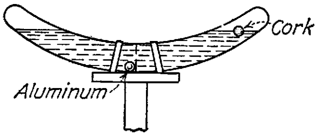

FIG. 44.—Model centrifuge; rotation causes cork to float on under side of water.

M-154. Whirling Bucket of Water. Perhaps not all students are acquainted with the time-honored trick of swinging a bucket of water in a vertical circle at arm's length without spilling the water.

M-155. Dime on Coat Hanger. A wire coat hanger hangs by its hook from the demonstrator's forefinger. A dime is laid on the middle of the horizontal bar of the hanger, and the hanger is then whirled in a vertical plane. With a little practice, this may be done without dislodging the coin.

M-156. Centrifugal Force Puzzle. Two steel balls in a narrow box are to be caught in two indentations *A* and *B* at opposite ends of the box (Fig. 45). One may try in vain to accomplish this result unless one knows how "to make a force act in two directions at once," simply by spinning the box about its midpoint *C*. One ball goes to each end instantly.

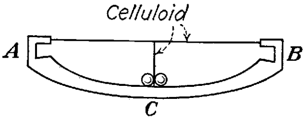

FIG. 45.—Centrifugal force trick—how can both balls be put simultaneously in the end pockets?

M-157. Loop-the-loop. A ball or small car rolls down a steep incline and thence safely around a vertical circle. The experiment offers opportunity for the discussion of dynamic equilibrium and the minimum speed for safe passage of the top point of the circle.

M-158. Centrifugal Governor. Models of Watt's regulator or centrifugal governor are commercially available or readily constructed.

M-159. Cream Separator. A spherical glass bowl containing mercury and colored water is whirled. The mercury forms an equatorial band around the bowl, with the water in two zones above and below it (and also nearer to the axis of rotation). But contrary to the almost universal expectation of students, the mercury represents the milk and not the cream!

M-160. Conical Pendulum. A ball on the end of a cord may be rotated mechanically at steady speed to show the characteristics of a conical pendulum. The experiment offers in simple form the chief elements of dynamical equilibrium, in which the bob is balanced under the force of gravity, the tension in the strings, and the dynamic reaction (centrifugal force). The apparatus of M-138 is essentially a conical pendulum in which the centripetal force on the bob is measured by the weight hung at the center.

MOMENT OF INERTIA

M-161. Racing Rollers. Prepare two tin cans, one with a lead core surrounded by wood, the other with a wood core surrounded by lead. Simple calculations will enable one to make the masses of these cans equal. In external appearance they are identical, and a balance will show that they have the same mass; but they roll with different accelerations down the same inclined plane. The can with the lead core always wins the race; but if both cans roll up a second incline, they reach the same height. It is evident, therefore, that both had the same kinetic energy at the bottom but that the can with the lead core had a relatively larger fraction of its kinetic energy in translation rather than rotation.

In a similar manner, balls, rings, and disks may be raced down the same incline, or their times of descent over a measured distance may be taken with a stop watch. The dependence of acceleration upon moment of inertia and the distribution of kinetic energy between rotation and translation may be discussed.

M-162. Winning Ball. All solid spheres, regardless of diameter or density, descend a smooth inclined plane with the same acceleration. A hollow sphere will always lose a race with a solid sphere. But if a hollow sphere, such as a tennis ball, is filled with mercury, it will always win against a solid sphere, because a larger proportion of its kinetic energy is energy of translation. A hollow steel roller filled with mercury will likewise accelerate faster than a solid cylinder, or even than a solid sphere. The mercury partakes only partially of the rotational motion of the body. A convenient mercury-filled roller may be made by sealing mercury in a test tube that fits snugly inside a brass tube whose end plates may be soldered. The mercury is thus prevented from amalgamating with the solder or the brass.

M-163. Weary Roller. A cylindrical tube partly filled with fine dry sand or powdered tungsten will exhibit the novel phenomenon of growing weary of rolling downhill. At the start, the contents of the cylinder are tamped into one end by setting the cylinder vertically. It then rolls freely, but as it rolls, some of the powder shakes down and retards the roller by internal friction. It may even stop on the incline. The inclined plane should not be too steep. One may match such a loaded cylinder with a hollow one of equal mass and dimensions, so that the loaded cylinder may either win or lose a race, depending upon the position of its load.

M-164. Falling Spool. A spool, such as is used for wire, is allowed to fall and unwind a thread held from above. It thus spins as it falls, and its acceleration is far less than the acceleration of gravity. A spool with heavy rim (large moment of inertia) and small axle will descend very slowly. The dependence of acceleration upon moment of inertia and torque may be made evident by the use of spools having different moments of inertia and axles of different diameters. To insure rotation about the axis of the spool only, it is well to support the spool by two threads wrapped around opposite ends of the axle. Although the descent is slow, the spool will rise again nearly to the height from which it was dropped if, after descending to the end of its threads, it winds up the threads in the opposite direction. The tension in the threads is less than the weight of the spool whether it is rising or falling, as may be shown by hanging the spool from a spring balance.

M-165. Rolling Spool. A spool consisting of two brass disks 3 in. in diameter connected by a 2-in. axle $\frac{1}{2}$ in. in diameter rolls on its axle down a narrow board. Its acceleration is very small because of the large moment of inertia. A rubber mat is placed on the lecture table at the point where the disk first makes contact, at which point the spool suddenly leaps forward as its kinetic energy of rotation changes in large measure to kinetic energy of translation. If a second incline is so placed that the spool rolls up it on the disks, it will be found that the spool rises (nearly) to the level from which it started, but in far less time.

M-166. Angular-acceleration Machine. The close analogies between angular and linear acceleration, torque and force, and moment of inertia and mass may be emphasized by showing the effect of increased moment of inertia upon angular acceleration when torque and mass remain constant. Two equal masses (1 kg) with setscrews are arranged to slide along a rod set perpendicular to a horizontal axle so that they may be fixed at various equal distances from the axis of rotation (Fig. 46). A mass M tied to a string wrapped around a pulley P of radius r produces a torque Q that gives the system an angular acceleration. The torque may be varied by changing either M or r. The decrease of angular acceleration α with constant torque but with increased distance of the masses m from the axis (increased moment of inertia I) is made evident. The device may be used, like Atwood's machine (M-110) in translational motion, to check the relation $Q = I\alpha$ for rotation. For example, by timing the descent of M over measured distances h with a stop watch, it is possible to show that t^2 is proportional to h, that the linear acceleration a of M is constant, and that the angular acceleration $\alpha = a/r$ is also constant. By using different accelerating masses or different radii, α may be shown to be proportional to the applied torque.[1] Next, by varying the distance d of the movable masses m from the axis, it may be shown that α is inversely proportional to d^2. Finally, if other masses m of the same figure but of different density are used, it may be shown that α is proportional to $1/m$, and therefore that $Q = kmd^2\alpha$.

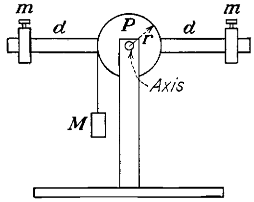

FIG. 46.—Angular acceleration of the masses m, m is produced by a measurable torque.

A ball-bearing bicycle hub serves as a good axle for this apparatus. If the cross rod is made of tubing and if the masses m are not placed too near the axis, it is possible to consider the moment

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[1] The torque must be corrected for the linear acceleration of M; $Q = Mr(g - a)$.

of inertia of the system as that of two point masses m at a distance d from the axis without introducing an error of more than 1 or 2 per cent. The apparatus may also be used for precise laboratory work.

M-167. Torsion Pendulum. The period of a torsion pendulum, $t = 2\pi\sqrt{I/Q_0}$, depends upon I, the moment of inertia of the suspended system, and upon Q_0, the torsion constant of the supporting wire. The suspended system may consist, as in M-166, of a cross rod carrying equal, movable masses whose distances from the support wire may be varied; the rod is rigidly clamped to a suitable torsion rod or wire. By changing the position of the masses by a known amount, and by timing the period of the pendulum both before and after shifting the weights, it is a simple matter to compute Q_0 and the moment of inertia from the two observed times and the known *change* of moment of inertia.[1] The torsion pendulum offers a standard method of determining the moment of inertia of an irregular object about any axis passing through its center of gravity.

M-168. Stable Axes of Rotation. A hand drill held vertically is used to rotate loops of rope or chain, supported by single wires. As the speed of rotation is increased, the object rotated assumes an axis about which its moment of inertia is maximum. The axis of rotation always passes vertically through the center of gravity of the object, even though the point of support may be some distance away. The rope or chain that otherwise hangs limp will, on rotation, take up a circular form in a horizontal plane. A stick supported from one end, or a hoop supported from a point on its periphery, will similarly revolve in a horizontal plane. If the demonstrator is adept in the use of a lariat, the same principle may be shown, or he may be successful in spinning a pie plate on the end of a stick.

M-169. Earth-Moon System. The combination of centrifugal torques that makes the chains and sticks of M-168 behave as they do may also be called upon to make a dynamic model of the earth and moon perform, demonstrating in particular how the system rotates about its common center of gravity. A 6-in. metal map globe of the earth is connected by a rigid tube 2 ft long to a

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[1] For details of this computation see, *e.g.*, Millikan, Roller, and Watson "Mechanics, Molecular Physics, Heat, and Sound," p. 351, Ginn & Company, 1937.

smaller sphere representing the moon (Fig. 47). The system is supported by a wire attached at the midpoint of the line of centers, not at the center of gravity. When the system is set rotating by a hand drill whose chuck holds the end of the wire, the two spheres revolve in horizontal planes about the common center of gravity of the system, which may be marked appropriately to make it visible. The demonstration may be made more impressive if small lights are arranged at the upper poles of the two spheres and a third light at the center of gravity. Flashlight dry cells are enclosed in the larger globe. The light at the center of gravity remains stationary while the two lights on the globes revolve around it in small and large circles.

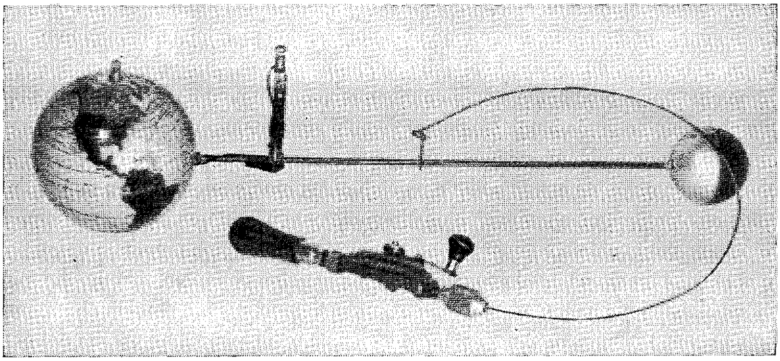

FIG. 47.—Earth-Moon system.

An apparatus consisting of two balls of different mass arranged to slide on a rod so that their common center of gravity coincides with the axis of rotation is commercially available, but its adjustment is critical.

CONSERVATION OF ANGULAR MOMENTUM

M-170. Constancy of Axis. In the absence of external torque, the angular momentum of a rotating body remains constant in both direction and magnitude. The constancy of direction may be shown by a spinning gyroscope supported in gimbals providing freedom of motion about three mutually perpendicular axes (M-192). If the axis of the spinning gyroscope is set in any chosen direction, it will maintain that direction while the stand on which it is supported is carried along a circuitous path.

M-171. A simpler apparatus consists of a 6-in. wooden disk supported by a loop of string passing through two holes drilled $\frac{1}{2}$ in. apart on either side of the center of the disk. The disk may be set into rotation in the manner of the familiar toy, by holding one end of the string loop in each hand with the disk midway between the hands. The string is twisted a few turns, and then by properly timed pulls the disk is set revolving at high speed about a horizontal axis, first in one direction, then in the other. At the instant when the disk comes to a stop before reversing its motion, the string is moved into a vertical position and again pulled. The disk reaches its highest angular speed when the cord is unwound, and while spinning it drops to the lower end of the loop. The loop is released at this end, and the spinning disk may then be swung around in almost any manner by the upper end of the loop without changing the direction of its axis, which remains vertical. This constancy of axis may be contrasted with the random motion of the disk when it is not spinning.

M-172. How the Tail Wags the Dog. The demonstrator stands upon the revolving stool and swings a baseball bat. The law of conservation of angular momentum requires that, as the bat is given angular momentum in one direction, something else must acquire equal angular momentum in the opposite direction. The result is not conducive to the maintenance of professorial dignity.

M-173. Rotary Action and Reaction. The periodic changes of angular momentum imparted to the balance wheel of a watch by its spring mechanism necessitate equal and opposite changes in some other body. This fact may be made evident by suspending a pocket watch by its ring from a nail filed to a sharp edge. If the period of the watch swinging as a pendulum from this nail is commensurate with the period of the balance wheel, a large amplitude of swing of the watch as a whole results. It is possible to bring the two periods to resonance by equipping the watch with an adjustable counterweight. The watch may also be supported in a horizontal position by a torsion wire perpendicular to its face, so that its period as a torsion pendulum agrees with the period of its balance wheel.

The same effect may be shown without the necessity of tuning by simply laying the watch face down on a smooth surface. The watch as a whole oscillates to and fro through a small angle with the period of the balance wheel. The motion may be made visible by reflecting a beam of light from a small mirror mounted perpendicularly on the back of the watch.

M-174. The axles of two machines of the type described in M-166 are connected by a spring from a window-curtain roller (Fig. 48). The spring is wound up by turning one machine in the proper direction; then both machines are released simultaneously.

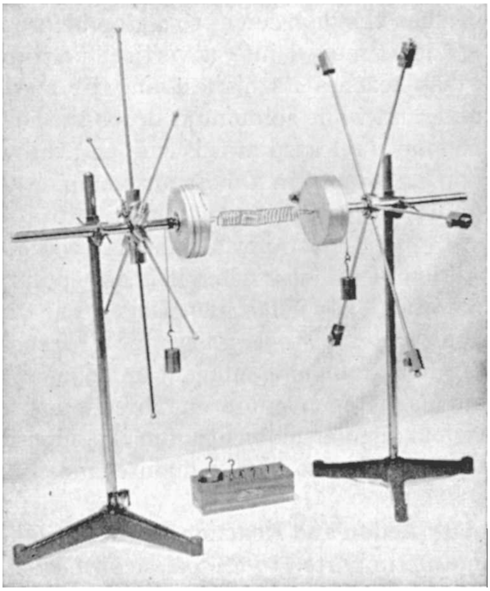

FIG. 48.—Rotary action and reaction.

Their senses of rotation are opposite, and their angular accelerations are inversely proportional to their moments of inertia.

M-175. How a Cat Turns Around in Midair. After attempting in vain to turn around while standing on the rotating stool, the demonstrator shows how it may be accomplished by variation of moment of inertia. He may turn himself completely around by extending his arms, executing a turn to the left, retracting his arms, executing a turn to the right, and repeating the process a number of times. A 5- or 10-lb weight held in the hands makes possible greater change in moment of inertia and hence allows one to turn around in fewer motions. It is important to show that even though the demonstrator succeeds in moving through an angle, he has had at no time a resultant angular momentum. That is, if he stops his arm and trunk movements at any instant, his net angular velocity should be zero.

The manner in which the cat executes a turn in midair may be discussed with the aid of a wooden model consisting of two pieces of 2-by-4 representing the hind- and forequarters of the cat, with appropriately designed hind and fore feet and a head, each with proper anatomical degrees of freedom.

M-176. Pirouette. The demonstrator stands on the rotating stool with arms extended and a 5- or 10-lb weight in each hand. He is given a small angular speed by an assistant, and as he lowers his arms his speed increases greatly. Since angular momentum is conserved, the decrease of moment of inertia necessitates higher angular velocity.

M-177. A mechanical apparatus for showing the pirouette effect is available commercially. It consists of two weights free to slide on a horizontal rod that is pivoted to rotate about a vertical axis (Fig. 49). The system is set into rotation with the weights at the ends of the rod, after which the weights are pulled toward the center by strings that pass over pulleys to a handle held at the center above the rod. As the weights approach the center, the angular speed of the system increases.

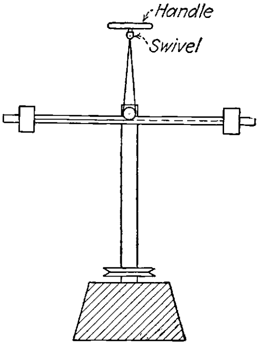

FIG. 49.—If the handle is raised while the horizontal rod is rotating, the weights are moved toward the center and the angular speed increases.

M-178. Bicycle Wheel and Rotating Stand. The demonstrator stands on a rotating stool and holds before him the front wheel of a bicycle equipped with handles on both ends of its axle. The wheel is held in a vertical plane passing through the axis of the stool. If the demonstrator spins the wheel, no rotation of the stool results; but if he then turns the spinning wheel into a horizontal plane, he will rotate in the opposite sense to the spin of the wheel. Turning the wheel over through 180° reverses his direction of motion; bringing it back to the vertical stops him. If he starts the wheel while it is held in a horizontal plane, he immediately acquires equal and opposite angular momentum. In this case, the equality of the two momenta is shown by the fact that the demonstrator stops rotating if he turns the wheel to a vertical plane while it continues to spin or if he stops the wheel while it is in a horizontal plane.

An electric motor with a flywheel mounted on the rotating stand with its axis vertical may show some of the same effects but less spectacularly.

M-179. "Quanta of Angular Momentum." It is interesting to show that angular momentum is a commodity that may be passed back and forth across the counter. The demonstrator stands at rest on the rotating stool. His assistant hands him a bicycle wheel spinning rapidly about a vertical axis. Nothing happens until the man on the stool turns the wheel over, whereupon he acquires angular velocity. He hands the wheel back to his assistant, who in turn rotates the axis through 180° and again hands it to the man on the stool. The angular velocity of the man is doubled when he turns the wheel over a second time. Additional "quanta" may be given to him by repeating the process. If, however, the assistant fails to turn the wheel over once but turns it over each time thereafter, the process becomes subtractive; the angular velocity of the demonstrator decreases to zero, and he starts in the other direction. In this manner, six or eight transfers of angular momentum may take place with a single spin of the wheel.

M-180. Baseballs or billiard balls may be thrown or caught at arm's length by the demonstrator as he stands on the rotating stool while his assistant stands on the floor. Each throw or each catch by the same hand increases the angular momentum of the demonstrator; he may decrease it or start himself in the other direction by catching the balls or throwing them with the other hand (other side of the axis). It would be less hazardous for the demonstrator to supply himself with a dozen baseballs that he may throw at a protected spot on the wall, thus illustrating by analogy the principle of the rocket car (M-119).

M-181. How to Work Up in a Swing—"Full-giant Swing." Pendular motion is a type of rotary motion and is, of course, governed by the same laws. How can one acquire large amplitude of motion in a swing without touching any external object? A simple answer to this question may be given by showing the work up of a croquet ball on the end of a long cord that passes through a screw eye in the ceiling or other rigid support (Fig. 50). It is well to attach to the cord a small wooden bead or other stop above the screw eye so that the ball cannot descend beyond a predetermined height, although it may be raised by pulling on the cord. It is possible to make the ball increase its amplitude of swing by properly timed pulls on the cord at *A* as it passes through the lowest point of its path. If the ball is raised a few inches each time it passes through the bottom point and is lowered to the stop each time it reaches an end of its swing, it will soon acquire a large amplitude. Reversal of this process will diminish the amplitude.

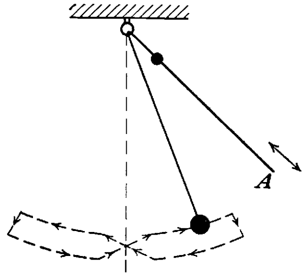

FIG. 50.—By pulling the cord at *A* so that the pendulum bob follows the path shown, its amplitude of swing is increased.

If, after the ball has acquired a large amplitude of swing, the cord is shortened sufficiently, the ball may be made to swing over the top in a full circle, just as the gymnast executes a "full-giant" swing. Naturally the screw-eye support must be located so as to afford sufficient space overhead if this last experiment is to be attempted.

M-182. A closer approximation to the actual case of a person working up in a swing is afforded by an electrically operated swing in which the moment of inertia is changed in proper phase by raising an iron weight with an electromagnet. The swing is supported by wires a meter long that serve as current leads to an electromagnet *M* (Fig. 51) situated on the seat. A movable iron core is kept above the magnet by a compression spring *S*. A key of the push-button type is mounted on the base of the apparatus to enable the operator to energize the magnet at will. The swing is given a small amplitude of motion, and then as it leaves the top of its swing, the magnet is energized, thus pulling the iron core down (representing the crouching position of the swinger). As the swing passes through the bottom point, the switch is opened, and the spring raises the iron core. The

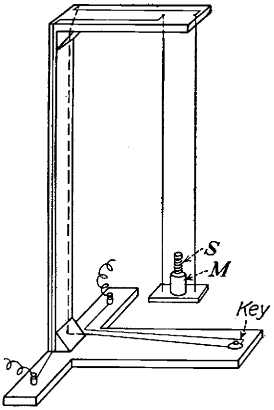

FIG. 51.—Model to illustrate work up in a swing.

amplitude may thus be built up as previously described (M-181).

M-183. Reaction Engines. The earliest type of steam engine (Hero of Alexandria, 120 B.C.) was a reaction type that may be represented by a cylindrical boiler pivoted on a vertical axle. Four tubes emerge from this boiler radially, and each has a nozzle directed tangentially in the same sense. When steam is generated in the boiler, some of it is forced out through the nozzles. The escaping steam exerts a reaction upon the steam in the tube, and the boiler is set into rotation.

M-184. A simple apparatus showing the reaction principle of the rotary lawn sprinkler consists of a tall tin can mounted on a float that is ballasted with lead to keep it upright. Two tubes emerge from the bottom of the can at opposite ends of a diameter and are bent in the same sense at their ends. When the can is filled with water, the jets from the tubes react upon the can to set it in rotation. If these two tubes are closed with stoppers and water is allowed to flow out of a straight radial tube, the can is driven sidewise. Instead of the float, a thread may be used to support the can.

The reaction of an electric engine upon a circular track free to rotate has been described elsewhere (M-123).

M-185. Transfer of Rotatory Kinetic Energy by Friction. A heavy wheel (gyroscope) is pivoted on a vertical axis in a balanced framework (Fig. 52) that is supported by a thread as shown. The wheel is given a spin and the framework released. As the wheel gradually slows down because of friction in its bearings, the framework as a whole takes up rotation with conservation of angular momentum.

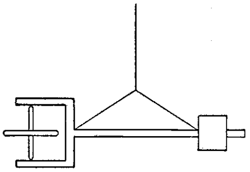

FIG. 52.—As the spinning gyroscope slows down, the framework takes up the rotation.

M-186. Paradox of Rotation. Whenever the moment of inertia I of a rotating system changes, one must examine carefully whether there is conservation of momentum ($I\omega$) or conservation of kinetic energy ($\frac{1}{2}I\omega^2$); for both $I\omega$ and $\frac{1}{2}I\omega^2$ cannot remain constant when I changes. For example, if a ball on the end of a cord is set rolling upon the lecture table and the cord is then allowed to wrap itself around a vertical rod, the ball describes a spiral path of decreasing radius. Its kinetic energy is conserved (barring frictional losses); hence, its angular momentum must decrease. In this case, angular momentum is transferred to the earth by reason of the torque applied to the rod by centrifugal force. By contrast, let the ball be given the same initial speed, but let the cord be shortened by pulling it through a hole in the lecture table. In this case, angular momentum is conserved in the ball, and the kinetic energy of the ball *increases*, even though the ball describes nearly the same path.[1] The additional energy comes, of course, from the work done in pulling the cord. The great increase in speed, both angular and linear, in this latter case is amazing.

GYROSCOPIC MOTION

M-187. Precession. Most of the phenomena of precession may be shown with a bicycle wheel equipped with handles on the ends of its axle (M-178). The wheel is given a spin by hand and one handle is slipped into a loop of string for support. When the other handle is released, the wheel precesses about a vertical axis while its own horizontal axis of spin slowly descends toward the vertical. If the precession is accelerated by pressure on the unsupported end of the axle in the direction of precession, the center of gravity rises. As the spin of the wheel diminishes, the wheel precesses more rapidly; or the precession may be made more rapid by adding a weight to the unsupported end of the axle. If the other end of the axle is supported in the loop of string, the sense of spin being unchanged, the direction of precession will reverse. From these simple phenomena, several of the important rules of gyroscopic motion may be worked out, such as the relation between directions of spin, torque, and precession and the relation between the magnitudes of spin, torque, and precession (M-188).

Since angular momentum is a vector quantity that may be conveniently represented by a vector parallel to the axis of spin, the combination of two angular momenta may be treated by the parallelogram law. Thus, whenever a gyroscope is acted upon by a torque tending to produce rotation about an axis perpendicular to the axis of spin (or of precession), the gyroscope will precess about a third axis perpendicular to the other two.

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[1] For a more refined method of insuring duplication of path in the two cases, see R. M. Sutton, *Am. Phys. Teacher*, **4**, 26, 1936.

M-188. Fundamental Precession Equation. The fundamental relationship $Q = I\Omega\omega$, where Q is the torque producing precession, I is the moment of inertia of the spinning gyroscope, Ω is the angular velocity of precession, and ω is the angular velocity of spin, may be shown with fair precision by applying a known torque to a balanced gyroscope (Fig. 53). The movable weight W with setscrew enables one to balance the top or to apply a measurable constant torque about a horizontal axis through the pivot. The angular velocity of precession is measured directly by timing the precession through one or more revolutions; the angular speed of rotation is measured by illuminating the spinning gyroscope with intermittent light from a neon lamp run on a 60-cycle circuit. Six spots are painted on the wheel at the corners of a regular hexagon whose center coincides with the axis.[1] The precession is timed for two or more speeds of spin to show that the product of these two angular speeds is constant (since moment of inertia and torque are also constant). The torque may be varied and the process repeated.

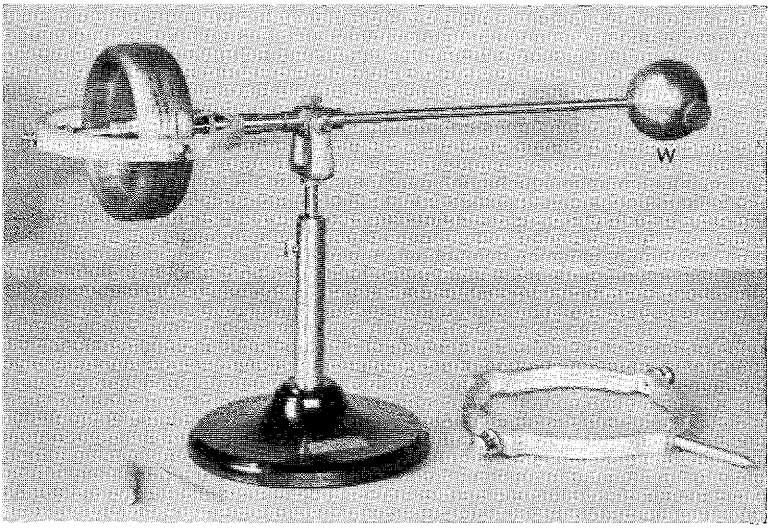

Fig. 53.—Gyroscope with movable torque weight W.

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[1] For example, with the neon lamp giving 120 flashes per sec on a 60-cycle circuit, the spots will appear to lie on the corners of a stationary hexagon at wheel speeds of 20, 40, 60, 80, . . . rps.

M-189. Balanced Gyroscope. The bicycle wheel with handles may be supported by loops of string tied to a crossbar that is hung by a single string from the ceiling, so that the wheel is in stable equilibrium whether spinning or not. In this case, the torque that produces precession may be applied by pushing the ends of the axle horizontally in opposite directions by the aid of two sticks held in the hands (Fig. 54). Since the axis of torque is now vertical, the axis of precession is horizontal and perpendicular to the axis of spin.

If preferred, the wheel may be mounted with its handles between vertical guides in a framework placed on the rotating stool. Since the stool revolves about a vertical axis and the wheel spins about a horizontal axis, the wheel will precess about another horizontal axis, and one end or the other of its axle will rise according to the direction of rotation of the table and of the wheel.

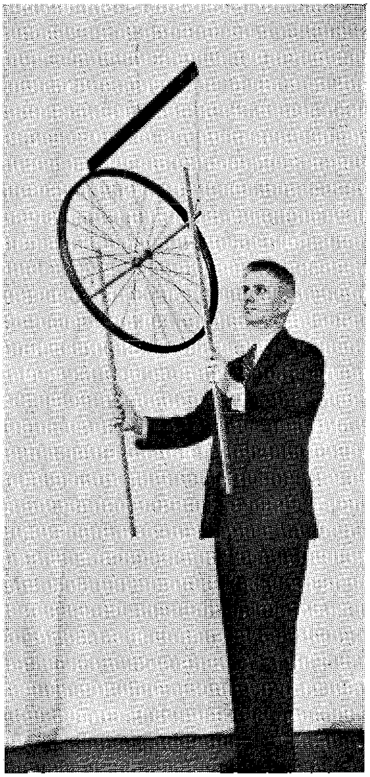

FIG. 54.—Applying a torque to a bicycle wheel gyroscope.

M-190. Walking Gyroscope. A heavy gyroscope is fastened rigidly to a stick a foot long, with its axis parallel to the stick. The stick hangs from two parallel horizontal rods which form

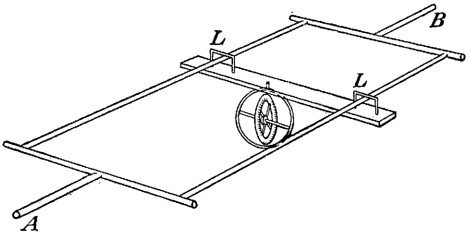

FIG. 55.—Walking gyroscope.

part of a cradle (Fig. 55) free to rock about a longitudinal axis *AB* parallel to the rods. The stick carrying the gyroscope is kept from leaving the rods by metal loops *L*. Now, with the gyroscope spinning, the stick may be made to progress in steps along the rod by simply rocking the cradle from side to side. When it reaches the end of the cradle, it can be made to return in a similar manner only by reversing the direction of spin of the gyroscope. It is evident that the motion is caused by a precession whose direction reverses each time the cradle is rocked. As the speed of the top diminishes, the speed of precession increases.

M-191. Maxwell's Top. A heavy wheel is arranged to spin about a pivot coinciding with its center of gravity. Thus there are no gravitational torques acting upon it, and it will spin steadily about any axis. If, however, a pencil is touched to the spindle of the top, the top precesses along the pencil, around the tip, and back along the other side. A helical coil of stiff wire may be placed over the spindle so that the spindle follows first the inner surface of the helix and then the outer surface (Fig. 56). The performance of the top is another application of the principles of precession: for as the spindle touches any object, it tends to roll; but this in turn produces a precessional torque at right angles to the direction of rolling, thus increasing the pressure between spindle and object and making the spindle "cling" to the object, even when it is of very irregular shape. A wire bent in a sinusoidal shape makes the top execute an interesting motion.

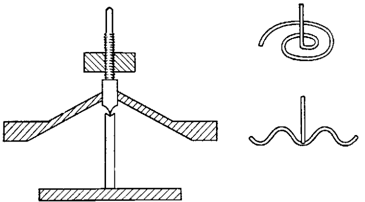

FIG. 56.—Maxwell's top and wire frames.

M-192. Gyrocompass. If a gyroscope is mounted in gimbals so as to have three degrees of freedom, then, in the absence of friction, it maintains an axis fixed in space, even when it is placed upon a rotating table or otherwise turned (M-170). But if it is deprived of one degree of freedom, the gyroscope will then turn until its axis of spin coincides in direction and sense with the rotation that is given to the top as a whole. Mount the top on the rotating stand, and observe the sudden change of the spin axis through 180° when the direction of rotation of the stand is reversed, even though the motion of the stand is almost imperceptible.

M-193. Airplane Turn Indicator. If a commercial-type turn indicator is not available, a model may be shown to illustrate the principle. A gyroscope is mounted in a yoke to spin about a horizontal axis *EF*; the yoke is free to rotate about a second horizontal axis *CD* perpendicular to the spin axis (Fig. 57). If the apparatus is turned about a vertical axis *AB*, the gyroscope precesses about the yoke axis *CD* in a direction depending upon the directions of turn and of spin. In the case of the airplane turn indicator, this axis *CD* is directed longitudinally along the fuselage.

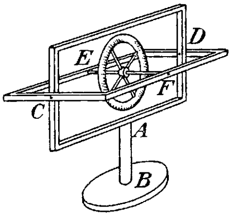

FIG. 57.—Model of airplane turn indicator.

M-194. Ship Stabilizer. There are several ways by which the stabilizing action of a spinning gyroscope may be shown. Some pieces of apparatus are available commercially (Fig. 58); others may be easily constructed. A model boat consisting of a horizontal piece of plywood with semicircular rockers is equipped with a gyroscope mounted with its spin axis vertical in gimbals so that it can precess about a transverse axis. As the boat rocks from side to side, the gyroscope precesses fore and aft. If the proper frictional torque is applied to the axis of precession,

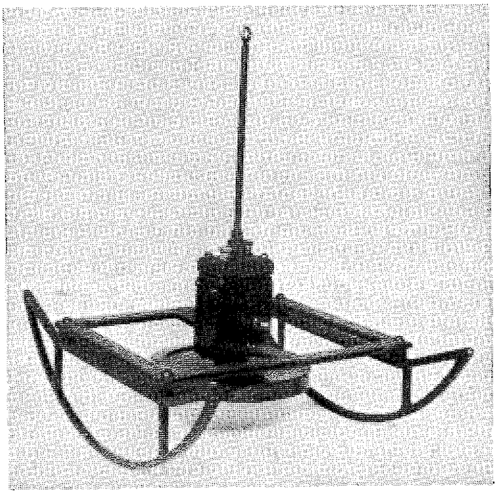

FIG. 58.—Model ship stabilizer.

the rocking is quickly damped. Too much friction prevents precession, and no stabilizing action results; too little friction allows precession to occur but produces no damping. A paper silhouette of a ship may be added to complete the model.

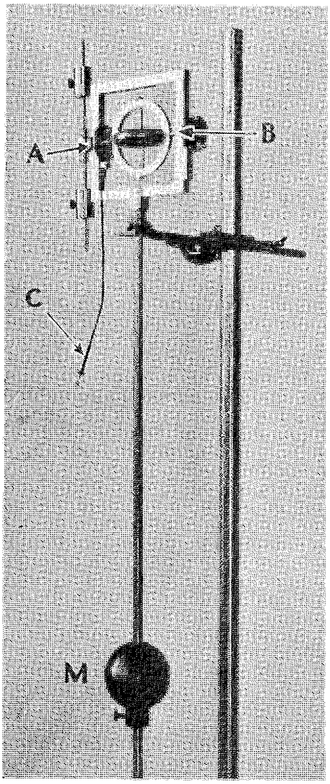

FIG. 59.—Frictional torque can be applied to the precession axis through the cable C to damp the motion of the pendulum.

M-195. The principle of the ship stabilizer may likewise be shown by a pendulum arrangement (Fig. 59) in which a gyroscope is held in gimbals so as to be free to precess about a horizontal axis AB in the plane of oscillation of the pendulum; the axis of spin of the top is vertical. The period about axis AB is controlled by the position of the weight M. Friction is applied to the axis of precession through a flexible camera release cable C while the pendulum is swinging.

M-196. A large-scale model is shown in Fig. 60. The demonstrator sits in the "boat," which is suspended from an overhead support so as to be free to swing like a pendulum about a longitudinal axis. He holds a handle connected to a motor-driven gyroscope by which he can control its precession. After the boat is set rocking, he may bring it to rest by four or five properly timed pumps on this handle. The converse action may likewise be shown, for the demonstrator may set himself into motion from side to side by rocking the gyroscope fore and aft.

M-197. Gyroacrobat. The stabilizing action of a gyroscope may be shown by the trapeze arrangement illustrated in Fig. 61. The top is rigidly attached to the center of a stick with its axis of spin perpendicular to the stick. Two cords are attached to the ends of this stick; midway along one cord, there is a light ring (wooden embroidery ring) through which the cord from the other end passes. The cords are thus crossed and tied to an overhead support. If the gyroscope is set spinning and then released when it sits on top of the horizontal stick as illustrated, it will ride in this unstable position for some time. However, if the gyroscope is released when the stick is rotated through 180°, no stabilizing action occurs, and the top promptly falls over. This experiment illustrates the most fundamental aspect of gyroscopic stabilizing, *viz.*, that there must be two degrees of instability about perpendicular axes.

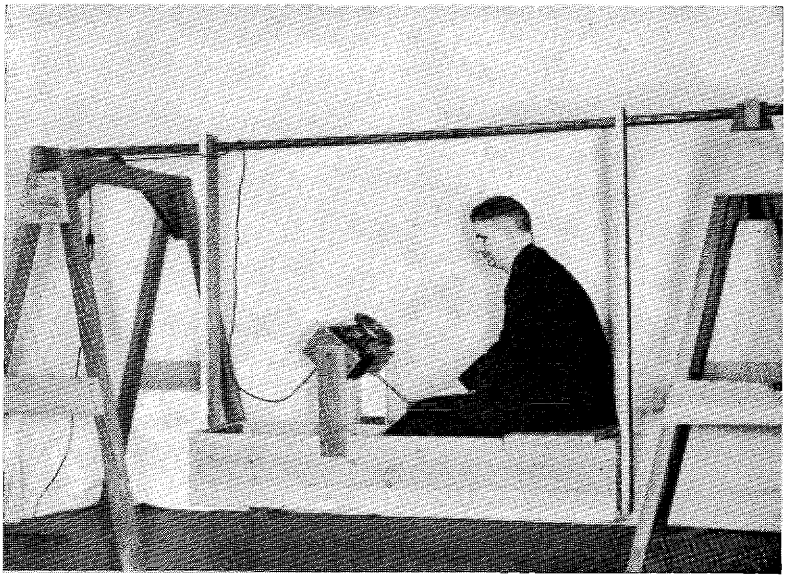

FIG. 60.—The demonstrator controls the rocking of the "boat" with the gyroscope.

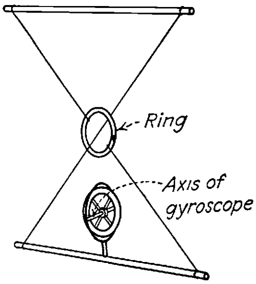

FIG. 61.—Gyroacrobat.

M-198. Gyro Bicycle Rider. A gyroscope is mounted with spin axis horizontal and with precession axis vertical on a model bicycle. A cylindrical weight W (Fig. 62) serves for the rider's head and also serves to accelerate the precession of the gyroscope when it is out of its position of unstable equilibrium, thus introducing a righting torque. The ends of the bar AB parallel to the spin axis are connected to the handle bars of the bicycle by a parallelogram linkage *ABCD* so that as the gyroscope precesses, the front wheel is turned.

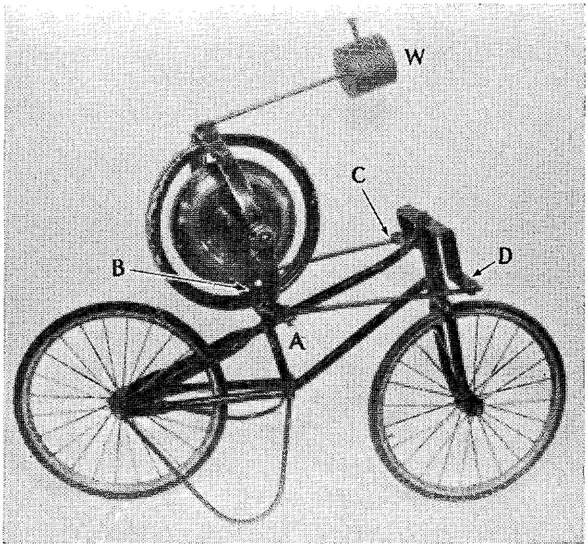

Fig. 62.—Gyro bicycle rider.

The bicycle and gyro rider may be pulled along the lecture table by a string, the righting action of the gyroscope lending a fantastic aspect of "intelligence" to the steering and balancing of the bicycle.

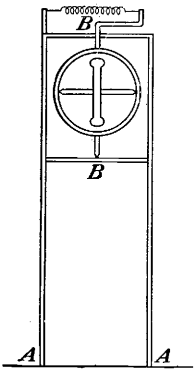

Fig. 63.—The spinning gyroscope stabilizes the top-heavy framework.

M-199. Gyro Drunken Man. Gyroscopic stabilizing may be shown by a top-heavy piece of apparatus that stands and teeters to and fro. A gyroscope is supported in gimbals with spin axis horizontal and precession axis vertical on top of a two-legged frame (Fig. 63). A lever arm attached to the gimbal frame allows the top to be turned about its axis of precession. A light spring accelerates the precession of the gyroscope whenever it departs from its position of unstable equilibrium. Thus there are axes of instability at right angles to one another, the axis of gravitational instability *AA* and the axis of instability introduced by the spring, *BB*. Hence when the top is spinning, the apparatus stands up and teeters about its position of unstable equilibrium so long as motion is allowed to take place freely about both axes of instability.

M-200. Monorail Car. There is on the market an excellent but expensive monorail car with two heavy flywheels that spin in opposite directions. The frames carrying these wheels are geared together so that they precess through equal angles in opposite directions. The ends of the axles of the two wheels are arranged to run on tracks so as to accelerate the precession of the tops whenever they depart from their middle positions. The car will run for several minutes on a steel cable or heavy rope.

An inexpensive monorail car may be made by converting a toy gyroscope into the stabilizing element. One of the gimbal pins of the top is threaded and screwed into a short piece of brass rod. The axis of precession *AB*, which runs transversely across the car (Fig. 64), passes longitudinally through this brass cylinder. Thus the axis of spin and the axis of precession are mutually perpendicular. The car is equipped with two wooden or metal pulleys *P* so that it can run down an inclined thread or fishline. After the top is set spinning, the car is released in an upright position, with the gyroscope axis vertical. It will run down an inclined thread the full length of the lecture room, teetering precariously from side to side. The car will right itself so long as the precessional motion of the top is allowed to take place freely. But if the amplitude of precessional motion builds up to the point where the frame touches the body of the car, the car instantly topples over.

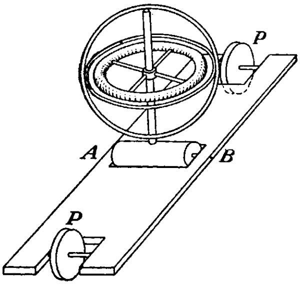

FIG. 64.—Monorail car.

M-201. Gyropendulum. A gyroscope is hung from one end of its spin axle by a string and is swung as a pendulum. Because of the precession introduced by rotation of the pendulum about its point of support, the gyroscopic pendulum bob executes a complex motion. If desired, a trace of this motion may be made by equipping the pendulum bob with a sand cone.

M-202. Tops. The manner in which the rolling of the peg of a top causes acceleration of precession and consequent rise of the center of gravity of the top may be demonstrated by means of a large mechanical top equipped with a rounded peg. The promptness with which the top rises and "goes to sleep" depends somewhat upon the character of the surface on which the peg rolls; if the top is spun on a sheet of glass, for example, the peg may slip on the glass, and the top fails to rise.

An egg-shaped top will rise and spin on end if given a sufficiently high velocity of spin. An oblate or prolate spheroid will likewise rise until its long diameter is vertical.

A hollow metal sphere with one hemisphere slightly heavier than the other will, upon being spun, precess until the heavy hemisphere is on top. If the ball is painted half black and half white, it is then evident that the equilibrium orientation of the ball when spinning is just opposite to that when at rest.

M-203. Compound Pendulum. A meter stick is supported on a pivot through a hole close to one end, so that it may be swung as a pendulum. The length of the equivalent simple pendulum is two-thirds the length of the stick, as may be shown by hanging a small ball by a string of proper length from the same support. The stick and simple pendulum swing with the same period. The point on the stick opposite the bob of the equivalent simple pendulum is the center of oscillation, which may be made the point of support without changing the period of the compound pendulum. It is an interesting problem for the advanced student to compute the position of the axis (one-fourth the length from one end) about which the stick will swing in *minimum* time.

In a similar fashion, the equivalent simple pendulum may be demonstrated for other objects supported as compound pendulums. Particularly interesting is the case of the hoop suspended from a point on its periphery and swinging as a pendulum in its own plane. The length of the equivalent simple pendulum is, in this case, equal to the diameter of the hoop. If, however, the hoop is swung in a plane at right angles to its own, the period is diminished. The dependence of period upon moment of inertia about the axis of support may thus be emphasized. Moments of inertia of irregular bodies may be computed by timing their periods when swung as pendulums about various axes.

M-204. Center of Percussion. The center of oscillation of a compound pendulum is sometimes also called its center of percussion, for it is the point at which a blow may be struck without causing reaction at the point of support. This property may be demonstrated in the case of the meter-stick pendulum (M-203) by supporting it on a matchstick instead of a nail. If the stick is struck a sharp blow with a hammer at the center of percussion (two-thirds the way down), there is no reaction on the point of support. But if it is struck above or below this point, the matchstick may be broken.

On a larger scale, the same phenomenon may be shown by dropping upon a movable fulcrum a 6-ft piece of 2-by-4 pivoted at one end with a piece of $\frac{1}{4}$-in. dowel rod. The dowel rod is broken unless the fulcrum is located beneath the center of percussion.

M-205. A heavy metal bar is suspended from one end by a long cord. Its center of percussion may be found by striking the bar horizontally with a hammer at various points. If the hammer strikes opposite the center of percussion, the upper end of the bar does not move. If the lower end of the bar is loaded, the center of percussion is thereby shifted farther from the point of support, and vice versa.

M-206. Falling Chimney—Free-fall Paradox. If a stick held in a vertical position with one end resting on the table is allowed to topple over, each particle of it describes a circular path. The center of percussion of the stick is the point that has the acceleration of a free particle along the path that it follows; all points beyond the center of percussion descend with accelerations greater than they would have if they were particles moving freely on their respective paths. In consequence of this, the stick finally reaches a certain position below which the vertical component of the acceleration of its end point exceeds the acceleration of gravity. For a uniform stick, this is at such an angle that $\cos^2\theta$ is greater than $\frac{2}{3}$, or θ is less than 35°(Fig. 65). A mechanical gadget may be made to illustrate this fact and to interpret the frequently observed backward buckling of the top of a falling chimney, which is caused, in part, by inertial reactions.

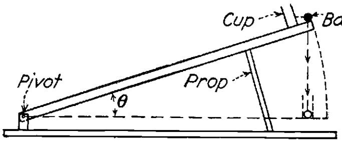

FIG. 65.—The falling stick drops away from the ball, which is caught in the cup.

Pivot a meter stick on a horizontal axis at one end so that it is free to rotate in a vertical plane. At a point about 85 cm from the axis, erect a light paper cup about 8 cm tall. Cut a slight indentation at the end of the stick to retain a steel ball when the stick is propped up at an angle of 35°. If, now, the prop is suddenly knocked out and the stick falls, the ball will fall into the cup even though the top of the cup started from a point higher than the ball. The action is quick and startling. A little trial will enable one to locate the proper position of the cup and of the prop to insure catching the ball. If a wad of paper is placed in the bottom of the cup, the ball will not bounce out to spoil the effect.

The same phenomenon may be shown more simply but less strikingly by supporting a meter stick in a horizontal position by the two index fingers. A wooden block is placed upon one end of the stick. If both ends of the stick are released simultaneously, the stick and block fall together. But if the stick is rested on one index finger and released at the end supporting the block, the stick falls away from the block as it rotates about the support. In other words, the end of the stick has a downward acceleration greater than the acceleration of gravity.

M-207. Foucault Pendulum. Many laboratories have arranged permanent mountings for a Foucault pendulum. A heavy ball on the end of a long steel wire is suspended in some convenient location, such as a stair well or elevator shaft. The ball is drawn aside by a thread, and after all motion is damped out it is set swinging by burning the thread. Its plane of oscillation may be observed to rotate slowly with respect to the floor of the room at a rate equal to the vertical component of the earth's rotation (15° sin λ per hr, where λ is the latitude) and in the opposite sense. The rotation will therefore be clockwise in the northern hemisphere as viewed from the point of support and counterclockwise in the southern. The magnitude of the motion is readily observable in the course of an hour in middle latitudes, provided the initial plane is marked.

There are many ways of supporting the pendulum and several ways in which the rotation of its plane of oscillation may be made evident in a few minutes after it is started.

A simple and effective method of supporting the pendulum so that it may be free to swing in any plane is to use piano wire clamped at the top and passed through a circular hole. of diameter slightly larger than that of the wire, in a horizontal steel plate rigidly mounted a short distance below the clamp. This obviates the necessity of using a more complex knife-edge or ball-bearing support.

The change in plane of oscillation of the bob may be shown by shadow projection of the motion of a small bead attached to the pendulum wire. The light should be placed as far as possible from the bead. The pendulum is started in a plane parallel to the direction of the light so that the shadow of the bead moves only along a vertical line on the wall. After a few minutes, the shadow shows an appreciable lateral motion that increases in amplitude as time elapses, and the component of motion perpendicular to the original plane of oscillation increases.

M-208. A compact, permanent arrangement of Foucault's pendulum is shown in Fig. 66. The pendulum bob is a 10-kg sphere hanging from the ceiling of the room by piano wire. The bob swings inside a wooden box with glass front and a slotted top to allow free motion of the wire. The motion of the bob is shown by projecting the image of a small ring of copper wire attached to the middle of the supporting wire. The optical system consists of an arc and a thin condenser lens that sends a beam, by reflection from a plane mirror, past the wire ring to an overhead screen inclined so as to be visible to all. The image of the marker is formed by a half lens placed close to the wire. On the screen are drawn two path lines at a small angle to one another. By adjustment of the optical system, the image of the marker is brought to the intersection of these lines when the bob is at rest, and the lines are so drawn that the image of the marker is on one of the lines when the bob is drawn aside in readiness to start. Thus, when the bob is released, the image of the marker moves back and forth along one path line. The light may now be turned off and turned on again after some computed interval, at which time the image of the marker should

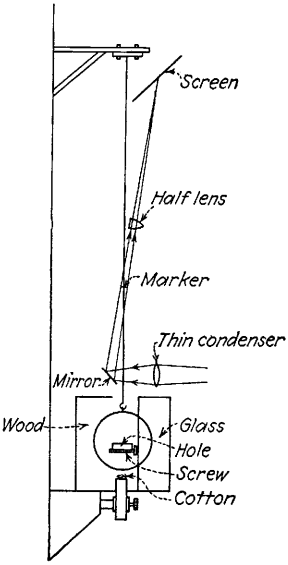

FIG. 66.—Optical arrangement for Foucault pendulum.

follow along the second path line. The position of the second line is, of course, optional. It will depend upon the length of time during which the pendulum is allowed to swing between observations, upon the inclination of the screen, and upon the local latitude. However, it is suggested that the lines be drawn at such an angle that the motion may be shown in twenty minutes or half an hour after starting the pendulum.[1]

M-209. Model Foucault Pendulum. The concept of relative motion that is involved in the rotation of the plane of oscillation of a Foucault pendulum may be illustrated (in exaggerated form) by mounting a simple pendulum on a rotating stool so that the pendulum bob in its rest position hangs over the center of the stool. The pendulum is set swinging, and the stool is rotated. The plane of swing of the pendulum remains unchanged relative to a stationary observer, but it would appear to an observer rotating with the stool to rotate in a sense opposite to that of the stool's rotation.

Interesting figures may be drawn if a sand pendulum is swung from the rotating support and allowed to drop sand on the rotating table. It is not essential to have the point of support coincide with the axis of rotation, provided that the speed of turning the table is not too great.

SURFACE TENSION

The successful performance of many of the following experiments depends upon the cleanliness of the glassware used and upon the quality of the soap solution.

Plateau's liquid is standard for blowing bubbles and making soap films. It consists of 1 part of oleate of soda, 40 parts of water, and 13 parts of glycerin. If prepared from free oleic acid, the formula is: 28.2 g oleic acid, 100 ml normal caustic soda solution, 300 ml glycerin, 1200 ml water. It is improved by adding 3 drops of concentrated ammonia to the above amount.

M-210. Ring Method of Measuring Surface Tension. One common method of measuring surface tension may be illustrated on a large scale by pulling a large brass ring away from the surface of water or other liquid. The ring is suspended from a light spring (Joly balance), and the extent of stretching the spring

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[1] Another method of showing rotation of plane of oscillation in a few minutes is described by S. R. Williams, *Pop. Astron.*, **40**, 256, 1932.

by surface forces may be measured roughly by calibrating the spring with known weights. The magnitude of the surface tension (*e.g.*, in dynes per centimeter) is then found from the measured force divided by $4\pi r$, where r is the radius of the ring.

M-211. Surface Tension of Mercury. The large surface forces present in mercury are shown by measuring the force required to pull a razor blade out of the mercury. It is essential that the mercury wet the blade. The blade is cleaned and dipped into copper sulfate solution to form a thin layer of copper on it and then into mercury in contact with nitric acid. When a coating of mercury has been made to adhere to the blade along one of its edges, the blade may then be dipped into mercury and pulled away from the surface on the end of a Joly balance spring.

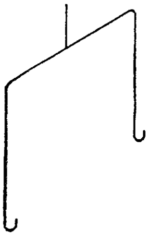

FIG. 67.—Frame for placing needles on surface of water.

M-212. Floating Metals. A needle rubbed between the fingers to make it slightly oily will float if laid carefully upon the surface of water. Razor blades, rings of wire, or paper clips are comparatively easy to float. Such metal boats may be sunk by touching the surface of the water with a drop of liquid soap. For placing needles upon the surface of water, a light wire frame (Fig. 67) is useful. It is essential that the metal objects be dry and slightly oily so that they are not wet by the water.

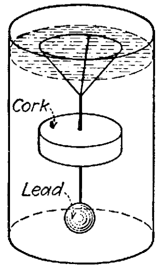

FIG. 68.—If the wire ring is pushed beneath the water surface, surface tension holds it there.

M-213. Submerged Float. A metal ring is arranged above a cork and lead float as shown (Fig. 68), so that the ring projects above the surface of the water. If the float is submerged until the ring is under water and then released carefully, the ring will no longer rise above the surface, but it will push the surface of the water up because of the buoyancy of the float. Again, a drop of soap placed upon the water will diminish the surface forces and allow the ring to break through.

M-214. Capillary Tubes. A set of connecting capillary tubes of various diameters will show that the capillary rise of water or other liquids is inversely proportional to the diameter. A similar set of tubes containing mercury will show capillary depression. (Project.)

M-215. Capillary Rise between Glass Plates. Two *clean* glass plates (30 by 50 cm) clamped together along one vertical edge and separated at the opposite edge by a thread or piece of wire are dipped into a vessel containing colored water. The water rises farthest near the clamped edge, and the curve formed by the surface of the water is a hyperbola, since the separation of the plates is proportional to the distance from the closed edge and the height of rise is inversely proportional to the separation.

M-216. Droplets in Tapered Tubes. A drop of water placed in a horizontal tapered glass tube open at both ends will move toward the narrow end of the tube. If the drop is displaced by air pressure toward the broad end of the tube, it will return when the pressure is removed. A drop of mercury in a similar tube will tend to move away from the narrow end, as may be shown by the fact that it requires a moderate air pressure to force the droplet farther toward the narrow end. (Project.) The concave meniscus of water and the convex meniscus of mercury are at the same time made evident.

M-217. Meniscus Effects. Two or three cylinders of wood or paraffin floated in a crystallizing dish containing water cling to the sides of the vessel. If more water is poured into the vessel until the meniscus is convex upward, the floating objects leave the edge and assemble at the middle of the water surface. (Vertical projection.)

M-218. Watertight Sieves. A box made of fine-mesh wire gauze dipped into hot paraffin so as to give it a thin coating will either float in a vessel of water like a boat or will hold water poured into it. However, the gauze boat may be sunk by dropping a single drop of water into it, or the gauze cup may be made to leak by touching its bottom with a wet finger.

A beaker of water may be covered with a piece of dry cheesecloth and turned upside down without the water escaping. It is not necessary to hold the cheesecloth, as it is held in place by atmospheric pressure just as a piece of paper would be.

M-219. Various Capillary Phenomena. If the surface of water in a beaker is covered with lycopodium powder, the demonstrator may dip his finger into the water without its becoming wet.

When a paintbrush is held under water, its hairs are separated as they are when the brush is dry; but when the wet brush is withdrawn, the hairs cling together. (Project.)

Pour water down a glass rod or a wet string.

Water poured into a box constructed from typewriter paper will cause the top edges of the box to be pulled inward because of the tendency of the surface of the water to assume the minimum possible area.

M-220. When Is a Glass Full of Water? A glass tumbler is filled level with the brim with water and placed before the projection lantern so that the class can see its image. The class may be asked to guess how many 4-penny finishing nails can be dropped into the tumbler without making the water run over. With care a surprisingly large number can be introduced, because the meniscus of the water may be made to stand well above the top of the tumbler. The time consumed in performing this experiment may tax the patience of the class. The same effect may be shown by adding water from a burette or pipette until the meniscus breaks.

M-221. Change of Surface Tension with Change of Surface Conditions. Place a thin layer of distilled water in a clean flat-bottomed dish on the vertical projector. If a drop of ether or alcohol is introduced at the center of the water surface, the water draws away quickly from the spot touched and leaves a large area covered only by ether or alcohol, which quickly evaporates.

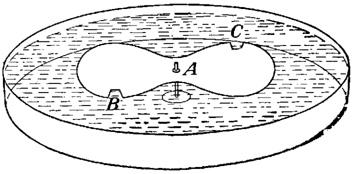

FIG. 69.—Bits of camphor at *B* and *C* cause the vane *A* to rotate.

M-222. A flat-bottomed dish containing distilled water is placed on the vertical projector. Lycopodium powder or chalk dust is sprinkled on its surface. If the surface is touched with a glass rod dipped in liquid soap, the powder is instantly swept out of the field of view. The effect is startling and unexpected. Ether vapor will produce temporary disturbances of the surface.

M-223. Bits of camphor placed on a clean water surface will dart about vigorously in random fashion over the surface because of unequal changes of surface tension produced where the corners of the camphor particles come in contact with the water. A little soap will quickly stop the motion. (Project.)

M-224. Camphor Mill. From a thin sheet of aluminum, cut a section like that shown in Fig. 69. At *A*, drill a hole large enough to slip over a long needle or piece of stiff piano wire. At *B* and *C*, cut back into the blades, and bend the cut pieces so that they will help to hold small pieces of camphor in the notches. Stick the long needle or piano wire into a flat cork weighted with lead so that it will sink in water and hold the needle vertical.

Place the aluminum blade with the camphor pieces over the pivot so that it rests on the surface of the water, supported by surface tension. The decrease in surface tension where the camphor touches the water causes an unbalanced torque that turns the blade round and round, away from the camphor.

M-225. Soap-propelled Boat. A matchstick or other light object that will float on water may be made to move about on the surface of water in a vessel as if self-propelled by simply rubbing one end of the stick on a cake of soap or by inserting a particle of camphor in a split at the stern of the "boat." (Vertical projection.)

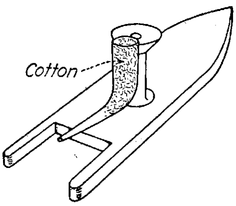

FIG. 70.—Alcohol-propelled boat.

M-226. Alcohol Boat. Cut out of thin wood a small flat boat of the design shown in Fig. 70. To this fasten a small spool, and to the spool fasten a glass tube drawn down to a capillary and bent with its small end just at the surface of the water at the stern. Put a small tuft of cotton in the glass "smokestack," and fill with alcohol, using an eye dropper. Place the boat on smooth water, and watch it move forward under the unbalanced forces of surface tension caused by the alcohol's contaminating the water at the stern. A shallow gutter pipe may be used as a stream bed with water running in at one end and out to the sink at the other. The boat will run "upstream" at a good rate.

M-227. Duco Boat. A drop of Duco household cement will dart around the surface of a vessel of water for several minutes. Two drops will "play tag" with one another in amusing fashion, avoiding one another and executing complex rotary motions as they progress. (Vertical projection.)

M-228. Mercury Amoeba. A globule of mercury about 1 in. in diameter is placed in a shallow crystallizing dish mounted on a vertical projector. The mercury is covered with 10 per cent nitric acid. If a crystal of potassium dichromate is placed near the mercury, the mercury starts to chase the crystal vigorously as soon as the red ring of dichromate diffuses out and reaches the mercury. (This behavior is probably due to change of effective surface tension by reason of electric charge on the surface of the mercury.)

M-229. Mercury Heart. A $\frac{1}{2}$-in. globule of mercury in a watch glass is placed on the vertical projector and covered with 10 per cent hydrogen peroxide to which is added a 1 per cent solution of sodium bicarbonate until a yellow film appears upon the mercury and breaks down regularly.

M-230. A $\frac{1}{2}$-in. globule of mercury in a watch glass is placed on the vertical projector and covered with 10 per cent sulfuric acid. A few crystals of potassium dichromate are stirred in with the acid. If the surface of the mercury is now touched with an iron wire, it contracts and pulls away from the wire; but the accumulation of a surface charge makes it again expand and touch the wire, whereupon it again contracts. A rhythmic pulsation begins, which may attain considerable amplitude and vigor for several minutes. The addition of more crystals of dichromate may restore the activity of the "heart."

M-231. Pulsating Air Bubble. Under an inverted watch glass beneath the surface of water in a vessel is imprisoned an air bubble $\frac{1}{2}$ in. in diameter. Alcohol is allowed to play upon the edge of the bubble through a bent capillary tube. The rate of flow is controlled by a screw clamp on a piece of rubber tubing connecting the capillary to a funnel (Fig. 71) and is adjusted until the bubble pulsates rhythmically.

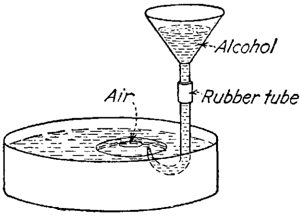

FIG. 71.—Pulsating air bubble.

M-232. Contraction of Soap Film. A soap film formed across the open end of an inverted funnel ascends toward the narrow end of the funnel, since the gain in gravitational potential energy is more than offset by the reduction of surface energy.

M-233. A U-shaped wire frame is equipped with a sliding wire link that closes the opening of the U. If a soap film is formed upon the frame, it tends to draw the wire link toward the closed end of the U. Hence, a small force is necessary to draw the link out, and upon release it will be drawn back suddenly by contraction of the film.

M-234. A cubical wire frame dipped into a beaker of soap solution shows several interesting examples of the contraction of soap films. By puncturing various parts of the film, different geometrical forms result; *e.g.*, a saddle-shaped surface may be made by puncturing the film so that it clings to the wire frame only along the four edges *AB*, *CD*, *EG*, and *FH* (Fig. 72). A film that touches nine of the edges but does not touch three mutually perpendicular edges that do not intersect, such as *EF*, *CG*, *BD*, contains a diagonal of the cube, in this case *AH*.

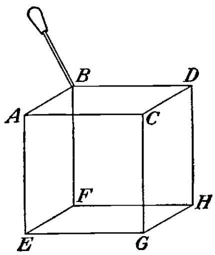

FIG. 72.—Wire frame for soap films.

M-235. Catenoid. A beautiful surface of zero curvature is formed by blowing a soap bubble over two independent concentric circular wire frames. The frames are then separated to make an elongated portion of the bubble between them, after which the bubble is punctured in such a way as to leave the film between the two frames. The film immediately takes on the form of a catenoid, in which there are two mutually perpendicular and equal curvatures at every point. As the frames are separated, the waist of the catenoid shrinks until a condition of instability is reached, whereupon the catenoid collapses at the center and two separate plane films are formed across the wire rings.

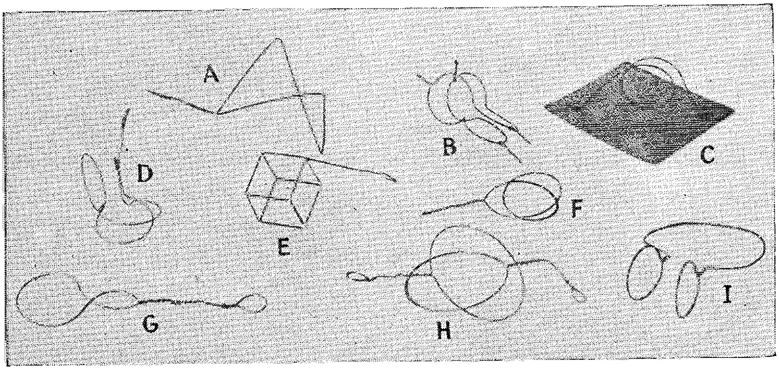

FIG. 73.—Wire frames for soap films to illustrate minimal surfaces.

M-236. Minimal Surfaces. Every soap film tends to reach a condition of minimum potential energy (M-232 to M-237) and is therefore (except for gravitational forces) a "minimal surface" for the contour presented by the frame. Many beautiful minimal surfaces are formed on appropriate wire frames. These surfaces furnish concrete illustrations of the solutions of a class of mathematical problems that has not yet yielded completely to rigorous treatment.[1] Figure 73 shows a few of the frames that form surfaces of special interest. The handles on frame *B* make it possible to change the film surface from one form to another by flexing the frame. Permanent films may be formed from collodion made somewhat thicker than the commercial grade by bubbling air through it and then allowing it to stand until all air bubbles have disappeared. The proper consistency must be determined by experiment. Care should be taken in withdrawing the wire frames from the solution. When the films have hardened, they may be sprayed with oil-base paint of any desired shade.

M-237. Ring and Thread. A wire ring has attached to it a loop of thread. It is dipped into a tray of soap solution so as to form a film within which the thread hangs limp. If the soap film is punctured inside the loop, the film on the outside of the loop contracts and draws the thread into a circle. The area of the empty circular loop is maximum for the periphery allowed by the thread; hence the area of soap film is a minimum.

M-238. Equidensity Bubbles. The demonstrator blows a soap bubble with a bubble pipe until it is 3 or 4 in. in diameter. The pipe is then connected to the gas supply, and the bubble further inflated until its mean density is that of the surrounding air. When detached from the pipe, the bubble may hover in the classroom for more than a minute. It is somewhat a matter of luck whether the bubble rises or falls after it is released from the pipe. However, it may be kept in control by gentle air currents from a piece of paper. It sometimes happens that the bubble is a trifle too heavy and begins to sink; but after it loses weight by evaporation its descent is halted, and it slowly rises to the ceiling.

M-239. Pressure within Bubble—Two-bubble Paradox. Two small bubbles are blown on the ends of a T-tube. By starting them at different times, it is possible to secure two bubbles of slightly different size. If the blowing tube is now closed and the bubbles are left to themselves, the smaller bubble will shrink because the pressure within it is greater than that within the larger bubble. Pressure within a soap bubble is greater than the

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[1] Courant, R., *Ann. Math.*, **38**, 679, 1937.

external pressure by an amount that is inversely proportional to the radius of the bubble.

M-240. Pressure within Bubble—Manometer Method. A slant water manometer that can be projected is connected by a side tube to the tube through which a soap bubble is blown (Fig. 74). The excess pressure within the bubble may then be shown, and its dependence upon the radius of the bubble verified.

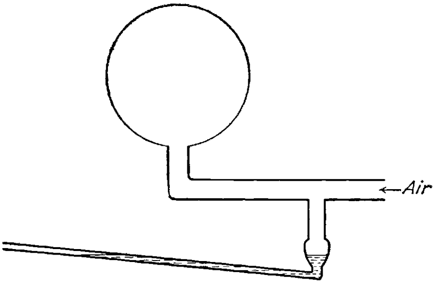

FIG. 74.—Measurement of pressure within soap bubble.

M-241. Mercury Bubbles. Air is blown into mercury under a dilute solution of ammonium chloride. Mercury bubbles rise to the surface and float for a second or so before bursting.

M-242. Drop Formation—Large-scale Model. A rubber balloon or membrane supported by a ring stand is filled with water until it is distended and constricted, very much in the form of an enormous drop.

M-243. Orthotoluidine Drop. Orthotoluidine has (at 24°C) the same density as water but does not mix with water. It is therefore an ideal substance for the formation of large drops. A large glass vessel (battery jar or aquarium) is filled with water into which salt solution is introduced to form a layer of slightly greater density at the bottom. Orthotoluidine is allowed to flow into the water very slowly through a burette, the lower end of which terminates in an inverted thistle tube (Fig. 75). A large drop of the dark liquid slowly forms and finally seals itself off by surface tension and descends toward the salt solution layer where it floats

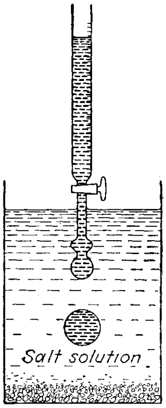

FIG. 75.—Large drops of orthotoluidine in water.

in virtually spherical form. The deliberate manner in which this event takes place allows the instructor time to point out the part played by surface forces in constricting the drop, the formation of Plateau's spherule, and the oscillation of the drop as it slowly acquires its spherical form. If several drops are allowed to form in this manner, they may be made to coalesce under water by dexterously leading small drops into larger drops with a glass rod. In this way, a single drop 2 or 3 in. in diameter may be formed.

M-244. Aniline Drop. Aniline likewise forms equidensity drops when the common temperature of the water and aniline is about 25°C. In this case, no salt solution is used. About 80 ml of aniline is poured into a beaker of cool water. The aniline rests on the bottom; but if the beaker is gently heated, the aniline rises to the surface of the water, where it cools in contact with the air, and returns to the bottom in large drops, which form slowly beneath the surface of the water.

M-245. Equidensity Drops. A beaker of water is prepared with a layer of salt solution in the bottom. On top of the water is placed a drop of mineral oil or olive oil. With an eye dropper or pipette 1 or 2 ml of colored salt solution is slowly introduced into the drop of oil. The oil forms a sac separating the salt water from the fresh, and the slightly greater density of the salt water causes a large drop to form. The drops are smaller than the drops of aniline or orthotoluidine previously described, but their appearance is the same, and the necessary materials are much easier to obtain. The drops thus formed in the oil membranes sink toward the bottom of the beaker but are stopped by the denser salt solution. They may remain intact for several hours or even for days.

M-246. Spherical Oil Drop. A flat-sided bottle is half-filled with water. Alcohol is poured on top of the water, and by means of a pipette a globule of oil is introduced at the interface, where it floats in nearly spherical shape, at a level where its density is equal to that of the surrounding mixture of alcohol and water.

M-247. Variation of Drop Size with Surface Tension. A rectangular glass tank is mounted on the table of the horizontal projector and filled with water. A funnel is connected by rubber tubing to a glass tube drawn down at the end and bent through 180°; a pinchcock on the rubber tube controls the flow of olive oil and 0.5 per cent oleic acid colored with Sudan III, which is contained in the funnel. When the glass tip is held below the surface of the water and the pinchcock is opened, drops of oil about 4 mm in diameter form and ascend to the surface. If $\frac{1}{10}$-normal solution of sodium hydroxide is added to the water, the surface tension of the oil is reduced so that the drops become much smaller; the reduction may be so great that no drops form and the oil issues in a continuous stream.

M-248. Mercury Droplets. The spherical appearance of small droplets of mercury on a flat surface may be shown by horizontal projection. The coalescence of mercury droplets in a watch glass is interesting to observe when viewed by vertical projection. Clear mercury is sprayed into distilled water, where it forms many small droplets, which do not coalesce. If a few drops of acid are added, the independent droplets coalesce in random fashion, setting the resultant drops into vigorous agitation.

M-249. Water Droplets. Small spherical droplets of water may be formed on a surface that water does not wet. A piece of glass or paper covered with smoke from burning camphor or kerosene may be used to hold the droplets for horizontal projection. Droplets of spray from an atomizer may be observed to bounce when they strike the soot-covered surface.

A tumbler is filled so full that the water surface is higher than the edge of the tumbler. Water from a pipette with a small opening is allowed to strike the surface at a small angle. Tiny droplets will be seen to roll across the water surface, and some may fall off the opposite edge. Occasionally a droplet may rest for a short time on the surface before coalescing with it. (Project.)

M-250. Tears of Wine. A 25 per cent solution of grain alcohol is poured into a watch glass, arranged for vertical projection. The liquid wets the glass but, as alcohol evaporates, surface tension increases and the remaining liquid forms into droplets that run down the glass.

M-251. Castor-oil Drop. Allow castor oil to flow slowly through a tube of small aperture placed in front of the horizontal projector. The viscous liquid forms a drop slowly.

A large colored drop of castor oil[1] is formed on the surface of water. A glass rod is thrust into it, and it is dragged down under

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[1] See footnote, p. 120.

the surface where it forms into a spherical drop, which clings to the rod and slowly climbs up it until it reaches the surface, where it again flattens out.

M-252. Liquid Drops on Surface of Liquid. Drops from the paddle of a canoe now and then roll for several feet on a still water surface. The same phenomenon may be shown by horizontal projection with drops of alcohol released upon the surface of alcohol in a dish. If the alcohol used for forming droplets is slightly colored, the progress of the liquid contained in the drop may be followed even after the droplet has coalesced with the body of the liquid. In this case, the colored alcohol produces a vortex ring as it proceeds beneath the surface.

M-253. Vortex Rings of Liquid. From a medicine dropper, allow a single drop of inky water to fall into a beaker of clear water from a height of just 1 in. If the water in the beaker has been standing for some time, so that its internal motions are small, a perfect vortex ring will be seen to dart down from the surface of the water, and its progress may be followed for 4 to 6 in. below the surface before it breaks up into a drapery of other rings and vortex filaments. If the water in the beaker is only about 4 in. deep, the ring will dart to the bottom and rebound. The height from which the droplets fall is critical. The best results are obtained with the height recommended. However, if the dropper is held so that the drop as it is nearly formed is "picked off" by the liquid surface, a slower moving vortex ring is formed (M-252). Other liquids show the same phenomenon, but water is the most convenient.

M-254. Liquid Jets. As liquid emerges vertically downward from a circular orifice, its jet contracts in diameter (vena contracta) both because of surface tension and because of the higher speed of the liquid as it descends. Project a portion of the jet above the point where it becomes unstable and breaks into droplets. Oil shows the effect better than water.

M-255. Necklace of Droplets. Arrange a jet of water directed at an elevation of 45° with the plane of its trajectory parallel to a screen or wall upon which the shadow of the jet may be cast. Adjust the water stream until the jet becomes unstable and breaks up into drops about 8 in. from the nozzle. If a tuning fork is sounded in the neighborhood of the jet or touches the stand holding the nozzle, the jet will become more stable and will break into regularly timed droplets of equal size at some point more remote from the nozzle. When the jet is illuminated with stroboscopic light (S-49) of the same frequency as that of the tuning fork, the stream of droplets will appear to stand still in a beautiful parabolic "necklace." If the speed of the stroboscope is altered, the beads appear to advance or return toward the nozzle.

M-256. Drop Formation Shown by Stroboscopic Illumination. A jet of water issues vertically downward from a small orifice situated just in front of the condenser lens of the projection lantern. An image of the stream is cast upon the screen by the projection lens. A cardboard disk containing a single hole is mounted at the focus of the condenser lens so that when the disk is rotated by a variable-speed motor, the light from the arc falls upon the screen intermittently. The vibration of the motor is sufficient to cause the jet to break into drops in a regular manner in synchronism with the illumination so that the image of the falling drop remains steady. By this method it is easy to demonstrate the process of drop formation, to show Plateau's spherule, and to show the oscillation of the newly formed drops.

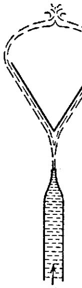

FIG. 76.—Water from the jet forms a bubble above the cone.

M-257. Bursting Water Bubble. A nozzle directed vertically upward sends a jet of water against the apex of a conical metal cap 1.5 in. in diameter (Fig. 76). The water forms a bubble above the cap, but if one drop of ether is allowed to fall on the bubble, it bursts, owing to the decreased surface tension.

M-258. Variation of Surface Tension with Temperature. Fatty and oily substances (*e.g.*, olive oil) form droplets on the surface of hot water but spread out over the surface as the water cools. The surface tension of both water and oil decreases with rising temperature, but the rate of decrease is greater for the water. Hence a temperature is reached where the two have equal surface tensions; above that temperature, the surface tension of oil is greater than that of water.[1]

M-259. Cohesion Plates. Two well-polished plate-glass surfaces show an appreciable cohesion for one another. Let each

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[1] For an extended treatment of surface films, see Wilson Taylor, "A New View of Surface Forces," University of Toronto Press, 1925.

student feel the pull necessary to separate a pair of clean dry plates 2 to 3 in. in diameter. If a drop of water is placed between the plates, the force perpendicular to the surface required to separate them is greatly increased, although the plates may be separated laterally with ease. (The effect in this case is undoubtedly due to both cohesion and atmospheric pressure.)

M-260. Cohesion of Water Column. In a long tube (2 to 4 m) closed at one end, water is boiled to drive out all dissolved air. The air is pumped out, and the tube is sealed off. The cohesion of the water column will support it against gravity and even against rather severe inertial shocks. Mount the tube firmly on a board for safe keeping.

M-261. Adhesion of Water to Glass. A piece of plate glass is supported by three strings in a horizontal position from one arm of a beam balance, as shown in Fig. 77. The plate is brought into contact with the surface of water in a beaker, after which the beaker is lowered a trifle to show that the water clings to the plate. Sufficient weights may then be added to the other arm of the balance to separate the plate from the water. Soft-rubber "vacuum cups" offer a convenient means of attaching the strings to the glass plate.

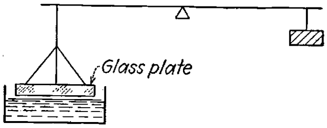

FIG. 77.—Adhesion of water to glass.

OSMOSIS AND DIFFUSION

Experiments on the diffusion of gases are described in A-54, A-55, and A-56, in connection with Kinetic Theory.

M-262. Diffusion of Liquids. A tall glass cylinder (graduate) is filled one-third full of a saturated solution of copper sulfate; the upper two-thirds is filled with water. A thin layer of oil over the water reduces evaporation. The cylinder is placed in a location where it may be visible but free from disturbance. The line of separation between the colored and uncolored parts of the liquid may be marked from time to time. In the course of time, the denser copper sulfate will be found to have diffused throughout the water.

M-263. Diffusion of Liquids. A glass tube 7 or 8 ft long and 1 in. in diameter is filled at its lower end to a depth of 6 or 8 in. with saturated copper sulfate solution, to which some crystals of copper sulfate are added. The remainder of the tube is filled with water, and the tube is then sealed. The date of filling is recorded, and the tube is kept in a vertical position as free from disturbance and from large temperature differences as possible. The progress of the color line may be marked from time to time. Such a tube may be kept for many years and exhibited annually. The slowness of the diffusion process will impress the students; for as time goes on, it may well be that the experiment was started before some of the students were born.

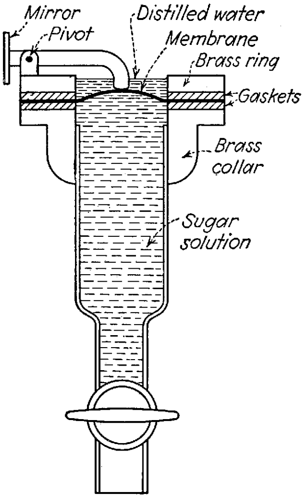

Fig. 78.—Optical osmometer.

M-264. Osmosis. A carrot or a beet into the stem of which is imbedded a glass tube securely sealed with paraffin is submerged in distilled water. The inward diffusion of the water soon causes water to rise in the tube well above the level in the vessel. Start the experiment, and keep it in evidence for several days, as the process is a slow one. Mark the level of liquid in the tube from time to time. The process is much more rapid if a cavity inside the carrot is filled with concentrated sugar solution before the top is sealed. In this case, the water may rise several feet in 24 hours.

M-265. An animal membrane or a vegetable parchment is sealed tightly over the end of a thistle tube connected to a long vertical glass tube. A saturated solution of sugar or salt is placed inside the thistle tube, and the tube is immersed in a vessel of distilled water. The level of liquid inside the thistle-tube system rises from day to day until a large pressure is attained.

Fill a small jar with dried peas, and add water until no air bubbles remain. Seal tightly. The jar will burst in a few days.

M-266. Optical Osmometer. Osmosis may be shown in the course of a lecture by means of an osmometer[1] in which the motion of the semipermeable membrane, caused by the increase in volume of liquid on one side of it, is made evident by an optical lever (Fig. 78). A brass collar is cemented to one end of a glass

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[1] Dwight and Kersten, *Rev. Sci. Instruments*, **5**, 130, 1934.

tube, the other end of which is fitted with a stopcock. The membrane is held between the collar and a brass ring by three screws; leakage is prevented by rubber gaskets. The optical lever consists of a bent rod, of which one end rests on the center of the membrane and the other, which carries the mirror, is pivoted at the edge of the ring. The tube is filled with the solution to be studied, and distilled water is poured into the ring above the membrane.

LIQUIDS AT REST

M-267. Distinction between Fluids and Solids. Fluids are distinguished from solids by the ease with which parts slide over one another. A fluid cannot permanently sustain a shearing stress. If a bit of asphalt or tar is smashed with a hammer, it splinters like glass, but if the chips are placed in a watch glass, they will gradually flow out in a few days into thin wafers. On the other hand, the sand in an hourglass flows readily through the neck but comes to equilibrium in a conical pile, which never flattens out.

M-268. Distinction between Liquids and Gases. Gases are readily compressible, but the compressibility of liquids is very slight. An empty bottle used as a hammer to drive a nail would break into pieces at once. A $\frac{1}{2}$-l reagent bottle with a ground-glass stopper, if filled *completely* with water (previously boiled to expel the absorbed air), may be used with impunity to drive an 8-penny nail up to its head in a pine plank. Since the glass is backed by an incompressible fluid, it cannot yield enough to break.

M-269. Incompressibility and Transmission of Pressure in Water. A large bottle is completely filled with water and closed with a tight-fitting stopper. If the stopper is struck a sharp blow with a hammer or the heel of the hand, the bottle will be shattered. *Caution:* Hold the bottle over the sink by means of a cloth wrapped about its neck.

M-270. Compressibility of Water. Many laboratories are equipped with a piezometer of the form invented by Oersted. However, a simpler type may be constructed to demonstrate the compressibility of water. A $1\frac{3}{4}$- to $1\frac{1}{4}$-in. pipe reducer has the hole in the small end turned out so that it fits a rather heavy piece of glass tubing about 18 in. long. One end of this tube is closed; the other is slightly flared to fit the pipe reducer (Fig. 79). This tube is firmly sealed to the reducer with marine glue. A second piece of tubing, about 1 ft long and of such a diameter as to fit inside the first tube, is closed at one end and is extended at the other by about 6 in. of capillary tubing with a bore of 0.5 mm. This second tube and capillary is completely filled with air-free water and inserted in the first tube, capillary end down. About 1 cm of mercury is placed in the bottom of the outer tube and the water content of the inner tube adjusted by gentle heating so that for the temperature at which the piezometer is to be used the mercury rises a short distance up the capillary.

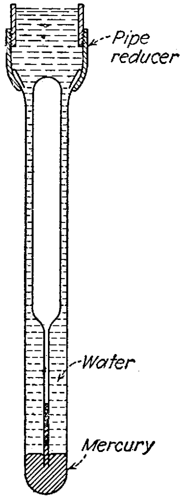


FIG. 79.—Compression of water within the inner tube is shown by the rise of mercury in the capillary.

The larger end of the reducer is equipped with suitable pipe fittings so that it may be attached to a convenient faucet by means of a piece of garden hose. The whole apparatus is mounted so that the top of the mercury in the capillary may be projected on the screen. In projecting the capillary, the condenser should focus the crater of the arc on the tube. The details of the capillary will then stand out clearly in a reduced field of view. The meniscus of the mercury will easily be seen to rise and fall as the pressure in the mains is turned on and off. Regardless of changes of volume of the outer tube due to change of pressure, the inner tube is of constant volume, because the pressure upon its walls is the same both inside and out. Therefore change in the volume of the water within the inner tube, caused solely by compression, is shown by change in the position of the mercury.

M-271. Pressure the Same in All Directions—Pascal's Law. The transmission of pressure in all directions throughout a liquid may be demonstrated by means of the pressure syringe. A spherical glass bulb blown on the end of a tube is perforated with equal small holes pointing in various directions on an equator, which is held horizontal. The tube is fitted with a piston and filled with water. This water squirts out equally in all directions when forced by the piston.

M-272. A modification of this pressure apparatus consists of a large closed metal container with several projecting tubes covered with rubber diaphragms. The container is filled with water, and no matter which of the diaphragms is pushed inward, all the others expand outward.

M-273. In another apparatus, three thistle tubes are equipped with light rubber diaphragms. The stems of the three tubes are held together with rubber bands to form a single central shaft. One tube is pointed downward, another is bent so that its diaphragm is vertical, and the third is arranged to face upward, with the centers of the three diaphragms in a horizontal plane. The diaphragms are expanded equally when the tubes and their stems are filled with colored alcohol to equal heights. When the thistle tubes are immersed in a vessel of water, the levels in the three stems are still equal. Instead of the three-tube apparatus, a single thistle tube with diaphragm may be arranged so that the diaphragm may be turned about a horizontal diameter.

M-274. Hydraulic Balance. A vertical glass tube 2 m long is connected to a rubber hot-water bottle, which is then filled with water. The level of the water in the tube is raised when a person stands on a board laid on the water bottle but the water does not squirt out of the top, as one might expect. The water rises to approximately the height of the person standing on the bottle, because his density is nearly that of water. Different individuals in the class may be "weighed" in this manner.

M-275. Pressure Depends on Depth. Tubes of various shapes rise from a common horizontal tube. Water is introduced, and the equilibrium level is shown to be the same in each tube.

M-276. Water pressure will hold a thin brass disk against the lower end of an empty glass cylinder held vertically in a vessel of water, but the disk falls off when water is poured into the cylinder until the level of water inside and out is the same.

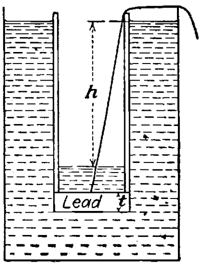


FIG. 80.—Pressure depends on depth.

A disk of lead (density 11.33 g per cm^3) 1 cm thick and of the same diameter as the outside of the hollow cylinder may be supported by water if the lead is about 11.5 cm or more below the surface of water outside the tube (Fig. 80). By means of a cord attached to its center, the lead may be held against the tube until sufficient depth is reached. A leather or rubber gasket between cylinder and lead will prevent leakage. When water is poured into the cylinder, the lead will fall off when the *difference* in levels h is less than 11.33 times the thickness t of the lead.

M-277. Pascal's Paradox. Two vessels made in the shape of truncated cones are identical in weight and size; one has the larger section of the cone for a base, the other the smaller. Both vessels are filled with water and placed on a platform balance. In spite of the greater force exerted on the larger base, the two vessels are in equilibrium. The bottoms of the vessels are then replaced by rubber diaphragms, and the vessels, supported by clamp stands, are lowered to the balance, so that its platforms support only the extended rubber. Equilibrium can no longer be maintained.

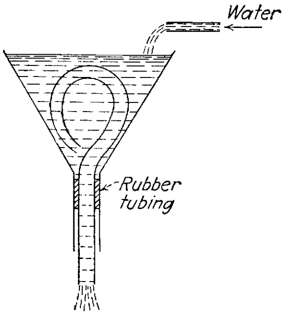


FIG. 81.—Intermittent siphon.

M-278. Siphons. A mechanical model of a siphon consists of a long flexible chain hung over a frictionless pulley. If once started, the chain will "flow" from one platform up over the pulley and down onto a somewhat lower platform.

An ordinary water siphon in air, as well as a mercury siphon in air or immersed in water, should be shown with the model. The intermittent siphon, or Tantalus cup, consists of an ordinary siphon contained within a vessel the sides of which rise higher than the top of the siphon (Fig. 81). This vessel is filled slowly, but when the water reaches the top of the siphon it begins to flow out. This action continues until the vessel is emptied; then the process repeats itself.

M-279. Self-starting Siphon. An ingenious device for starting a siphon by entrapped air is shown in Fig. 82. A bent tube bears near its end O a side arm B and a 25-ml bulb A with a small hole H. When the bulb is submerged in water, air is forced out through B by the entrance of water at H, and a column of air and water emerges from C.

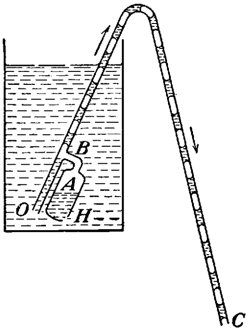


FIG. 82.—Self-starting siphon.

As the last of the air is expelled from the bulb, the siphon flow becomes steady.

M-280. Fountain Siphon. A delightful variant of the ordinary tube siphon may be made by connecting a flask and two tubes as shown in Fig. 83. When the level of water and the air pressure within the flask are properly adjusted, a steady stream of water shoots from the narrow orifice at *C*. To start the siphon, the flask is first filled with water and then inverted in the position shown. Air is admitted, little by little, through tube *A* by removing it momentarily from its reservoir, until the fountain runs freely. The greater the difference in level between *A* and *B*, the more vigorously does the fountain play. If the flow of water is stopped at *B*, the fountain continues to run for a time because of reduced pressure within the flask. It ceases to flow when the sum of the air pressure within the flask and the hydrostatic pressure above *A* is equal to atmospheric pressure.

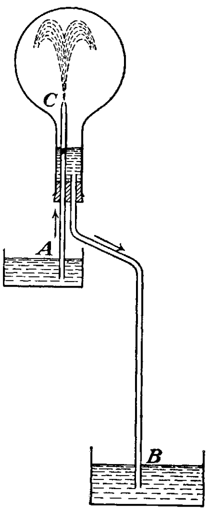


FIG. 83.—Fountain siphon.

M-281. Pressure Measurement in Siphon. A siphon is prepared from two bent glass tubes and a T-tube connected by rubber tubing at *A* and *B* (Fig. 84.) A mercury manometer is attached to the T-tube to measure the pressure in the upper part of the siphon. It shows clearly that the pressure here is less than atmospheric when the siphon is in operation. If the flow is stopped at *B*, the manometer measures pressure $H_1 - h$; if at *A*, $H_2 - h$; whereas if the siphon is running, the manometer shows an intermediate pressure that approaches $H_2 - h$ as a limit if large tubes are used to reduce loss of head due to friction. (Here h is a correction for pressure due to the water column between T and the surface of the mercury.)

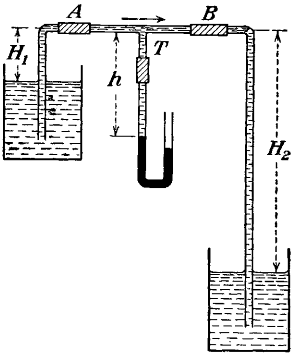


FIG. 84.—Siphon equipped with manometer for pressure measurement.

M-282. Pumps and Presses. Glass models may be projected to show the action of the valves of suction and force pumps. A hydrostatic press sufficiently large to break a piece of 2-by-4 timber, or a hydraulic jack fitted with a platform to lift several students, emphasizes Pascal's law of transmission of pressure.

M-283. Archimedes' Principle. A metal or weighted wooden cylinder is turned to fit closely within a cylindrical container or bucket of the same height. After the class has been shown the closeness of the fit, the cylinder is hung beneath the bucket from one arm of a balance. Weights are added to the other side until equilibrium is obtained. Now a vessel of water is brought up around the suspended cylinder until it is completely immersed. Equilibrium is destroyed but may be restored by filling the bucket to the brim with water, thus showing that the buoyant force on the cylinder is equal to the weight of water it displaces.

M-284. Another manner of demonstrating Archimedes' principle is to use a can equipped with a spout to carry off overflow. The can is filled to the spout and brought to equilibrium on a balance. An object *supported externally* by a thread is immersed in the water, and the displaced water is caught in a beaker standing on the table. The balance equilibrium is maintained, showing that the weight of the displaced water is equal to the buoyant force, whether the object sinks or floats. If an overflow can is not used, then weights equal to the buoyant force must be added to the other arm of the balance to maintain equilibrium.

A vessel of water is balanced on a platform scale, and the class is asked what will happen to the equilibrium if the instructor dips his hand into the water without touching the sides of the vessel.

M-285. Floating and Density. A tall jar is filled with liquids of varying density: mercury, carbon tetrachloride, water, cottonseed oil, gasoline. Objects of appropriate densities float at the interfaces: iron, bakelite, walnut, soft wood (shellacked), and cork.

Place an egg at the bottom of a tall jar filled with water, and throw a handful of salt into the water. Sufficient salt dissolves to float the egg part way up.

M-286. Hydrometers. The behavior of hydrometers may be shown by floating them in liquids of different densities. A constant-weight hydrometer sinks to different depths; a constant-volume (Nicholson) hydrometer requires different weights to sink it to the fiducial point. A Mohr-Westphal balance may be shown by shadow projection.

M-287. Instability in Flotation. A metal boat floats in a tub of water. Within the boat is an automaton so equipped with springs that he rises from a sitting to a standing position in response to a pull on his fishline. The springs *E* and *H* (Fig. 85) are extended in the sitting position. A pull on *A* turns *B* to hit the arm *C* and to release the catch at *D*. Rotation then takes place about the knee *K*, until the lever *I*, pivoted at *F*, strikes the pin *J*. Then the catch at *M* is released, the trunk of the body is free to turn about *G*, and the spring *H* pulls the body into an upright position, whereupon the boat and its occupant upset. Critical adjustments may be made by varying the weight of the keel of the boat.

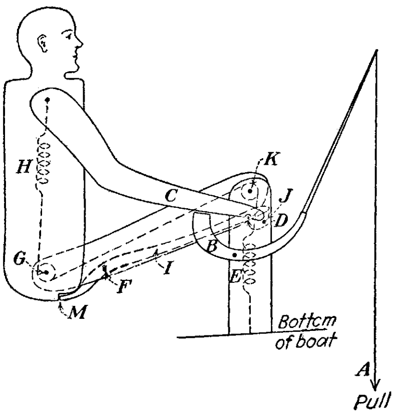


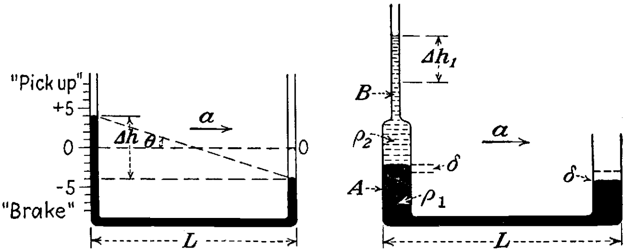
FIG. 85.—When the line is pulled the mechanical man stands up and the boat upsets.

DYNAMICS OF LIQUIDS

M-288. Inertia of Liquids. Two glass tubes about $1\frac{1}{2}$ in. in diameter and $\frac{1}{2}$ ft long, with their ends smoothed in a flame, are provided with corks. In one is placed a cylindrical piece of cork of diameter about $\frac{1}{4}$ in. less than the internal diameter of the tube. In the other is placed a cylindrical piece of lead of about the same shape. One of the corks in each tube may be of the ordinary tapered sort; the other cork should have a small shoulder that can be pressed down so as to bear against the end of the tube. After the two tubes are filled with water and corked, they are laid horizontally on the lecture table. A sharp blow of a mallet on the shouldered stopper of the tube containing the cork cylinder will cause the cork to shoot forward, while a blow similarly given to the stopper of the tube containing the lead cylinder will cause the lead to move backward in the tube.

M-289. Liquid Accelerometers. The inertia of a liquid may be utilized in various manometers for measuring linear acceleration, either vertical or horizontal (M-108), centripetal acceleration and speed of rotation (M-150), or angular acceleration (M-166).[1]

In simplest form for the measurement of horizontal acceleration, the manometer consists of a U-tube filled with mercury or water (Fig. 86*a*). The inertial reaction due to acceleration of liquid in the horizontal arm of the U is equalized by a difference in pressure in the two vertical arms, so that $\Delta h = La/g$, which is independent of the size of tube or density of liquid. The sensitivity of the device depends primarily upon the horizontal length L.



(a) (b)

FIG. 86.—Liquid accelerometers: (*a*) simple form; (*b*) multiplying form.

For lecture purposes (M-108), it is desirable to increase the sensitivity of the accelerometer without increasing its length. This may be done by making the manometer as shown in Fig. 86*b*. The horizontal arm is filled with mercury, which extends into the vertical arm a short distance. The area of cross section of one arm is reduced from A to B at some point above the interface between mercury and colored water in that arm. Thus the inertia of the horizontal mercury column is equalized in large part by the change in level of water in the constricted arm of the U. The relation between change in height in this arm alone and the horizontal acceleration is then given by

$$\Delta h_1 = \left(\frac{\rho_1 A}{2B\rho_1 + (A - B)\rho_2}\right) \cdot \frac{La}{g},$$

where ρ_1 and ρ_2 are the densities of the heavy and the light liquids respectively. It will be observed that the relationship between a and Δh_1 is linear and that the instrument may be cali-

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[1] SUTTON, R. M., *Am. Phys. Teacher.* **3**, 77, 1935.

brated linearly along the tube *B* directly in feet per second per second or other convenient unit, solely from a knowledge of the geometrical and density factors involved.

M-290. Water Hammer. A water-hammer effect can often be shown by suddenly shutting off the faucet at a local sink.

A sharp metallic click is heard when water strikes the end of a glass tube from which air has been removed. A tube 1 ft long and 1 in. in diameter is closed at one end and drawn down at the other. Two or three inches of water are placed in the tube, and it is connected to a vacuum pump with an efficient drying system. Pumping is continued until the water boils freely at room temperature. The small end is then sealed off. The cryophorus tube (H-67) shows the effect nicely.

M-291. Hydraulic Ram. Essential to the positive action of a hydraulic ram is a long feed pipe in which the water flows with high velocity. This pipe should be 15 to 20 ft long, with a drop of 3 or 4 ft. If made of $\frac{1}{4}$-in. lead tubing, it may be coiled beneath the reservoir *R* (Fig. 87). The valves are steel balls; valve seats at *A* and *B* are made by driving the balls against ends of the lead tubing. The ball at *A* is retained by a cage of four wires within the air chamber *C* (a 1-l spherical flask). The outlet *J* may be higher than the reservoir *R* and for convenience may be situated above *R* to help replenish the water supply. The outlet *D* leads to the sink.

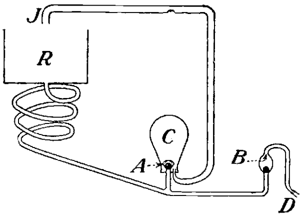

FIG. 87.—Hydraulic ram.

M-292. Ball on Jet—Bernoulli Effect. A ping-pong ball may be supported on a jet of water at a height varying from 3 to 15 ft. The jet must be as nearly vertical as possible. A piece of glass tubing drawn down to a narrow opening, not over 1 mm in diameter, serves as a nozzle. A large basin is necessary to catch the spray if great heights are employed.

Compressed air will serve as well as water. It is possible to support a ping-pong ball for several seconds on air from the lungs, blown through a nozzle 4 mm in diameter. When compressed air is used, the jet may be inclined at a considerable angle. It may be made to support a heavy rubber ball or even an ordinary screwdriver.

Instead of compressed air, a vertical jet of steam may be used. It is produced by boiling water in an ether can that is fitted with a tight cork containing a glass nozzle drawn down to a small opening.

M-293. Ball in Funnel. A ping-pong ball will be supported in the narrow end of an inverted funnel through which a jet of water or air is streaming downward.

M-294. Bernoulli Apparatus. A glass tube $1\frac{1}{2}$ or 2 in. in diameter is gradually tapered toward the center to about $\frac{3}{8}$ in. Several vertical side tubes are attached at points of different diameter along the tube. When water is sent through the tube, the effect of varying velocities of flow is shown by the different heights to which water rises in these tubes. Colored floats help to make the various water levels visible.

M-295. Card-and-spool Experiment. A pin is stuck through the middle of a card and is inserted in the hole of a wooden spool to prevent lateral motion. When one blows through the hole, the card does not fly off but presses against the spool. If a large metal disk with central hole connected to a large compressed-air outlet or tank of carbon dioxide gas is used, a similar disk is held to it with sufficient force to support a heavy weight.

M-296. Dependence of Pressure on Velocity in Air Stream. Hang two sheets of paper from the sides of a stick so that they are separated 2 to 10 cm. Blow between them, and observe how they are drawn together instead of being forced apart.

Alternatively, hang two pendulum balls a short distance apart, and blow between them. The balls immediately collide with one another. If metal balls supported by wires from a nonconducting support are connected with a lamp and 110-v supply, the collision of the balls may be made evident by a flash of the light.

M-297. Curve Balls. A throwing tool for shooting curve balls may be made from a 15-in. length of 2-in. mailing tube lined with sandpaper or from a smaller tube cut in two lengthwise and similarly lined. Ping-pong balls may be thrown from this tube with high spin velocity so as to describe "ins," "outs," "drops," or "upshoots," depending upon the manner of throwing. The ball always curves in the direction toward which its leading point is spinning.

M-298. Lift on Rotating Cylinder—Bjerknes' Experiment. Three feet of wide cloth tape is wrapped around the middle of a paper mailing tube 2 in. in diameter and 1 ft long. The free end of the tape is attached to a chain of stout rubber bands fastened to the top of the table. The tube is laid on the table so that the tape will unwind from the bottom. When the tube is pulled aside horizontally and released, the sudden unwinding of the tape gives it high linear and spin velocities. If the speed is high enough, the tube will describe a vertical loop (Fig. 88).

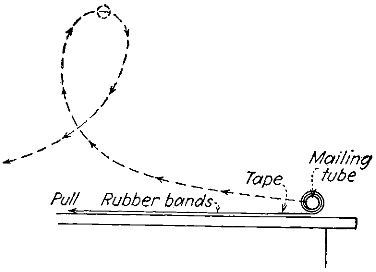

FIG. 88.—Bjerknes' experiment.

M-299. Autorotation. An 8-in. piece of half round is carefully balanced on an axle *AB* (Fig. 89). An air stream from an electric fan strikes the curved face of the stick. The stick will rotate at high speed in *either* direction when once given a start (see S-142).

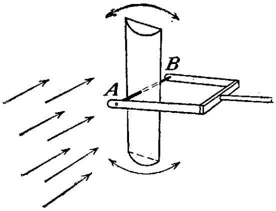

FIG. 89.—The stick is kept rotating in either direction by an air stream.

M-300. Flettner Rotor Ship. Upon a light car, arranged to run on rails with as little friction as possible, mount a light motor with its axis vertical. On an extension of this axis, mount a paper drum rotor about 1 ft high and 6 in. in diameter. The ends of the rotor should be fitted with flanges that extend about 1 in. beyond the cylindrical surface (Fig. 90). With this drum rotating rapidly, make the "ship" sail back and forth along the track by a stream of air from an electric fan directed at right angles to the track. The direction of the "ship" may be reversed by reversing either the direction of the wind stream or the rotation of the drum. With the rotor at rest, the breeze is unable to move the car.

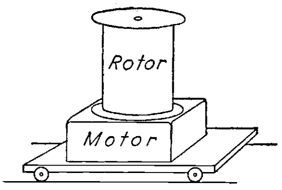

FIG. 90.—Model of Flettner rotor ship.

The same effect may be demonstrated less spectacularly by balancing a motor, driving a horizontal drum, on one pan of a platform balance. A stream of air will raise or lower the platform, depending upon its direction and the direction of rotation.

M-301. Lift on an Airfoil. Hold a sheet of 8- by 11-in. typewriter paper by one edge so that the surface near that edge is nearly horizontal, while the rest of the sheet hangs limp below. Blow across the top of this edge, and observe how the whole sheet rises to a horizontal position.

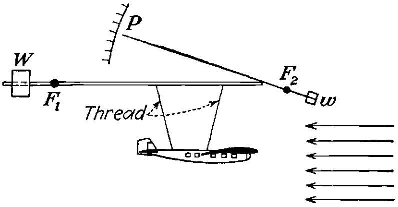

FIG. 91.—Balance to show lift on model airplane.

M-302. A model airfoil or airplane is balanced at F_1 as shown in Fig. 91. When a horizontal wind stream from an electric fan strikes it, the balance is upset as shown by motion of the pointer P balanced at F_2. The angle of attack may be altered by changing the lengths of the supporting threads.

If preferred, the airfoil may be suspended by two rods or by four wires as a pendulum, to show both lift and drag for various angles of attack.

M-303. Pressure on Surfaces of Airfoil. The pressure over the convex surface of an airfoil in a wind stream is less than that over the concave surface, as may be shown by the apparatus in Fig. 92. The airfoil is a hollow container, closed except for small holes at A and B. A slant manometer is attached at C. When the airfoil is placed in the wind stream from an electric fan, the slant manometer shows the pressure at A when B is

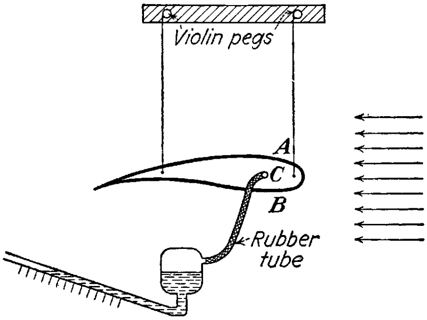

FIG. 92.—Pressure on surfaces of airfoil.

closed and A is open, and at B when A is closed and B is open. Variations of these pressures with angle of attack may be shown by tilting the airfoil.

M-304. Aspirator and Atomizer Effect. A 6-in. length of glass tube, $\frac{1}{2}$ in. in diameter, has a constriction in the middle to a diameter of about 0.1 in. Water is run through the tube to a vessel into which the lower end of the tube dips, to reduce noise and splashing. Reduced pressure in the constriction causes dissolved air in the water to be freed and even makes the water boil. The vapor condenses below the constriction, and the hissing sound that results can be heard throughout the room.

The side tube of a water-faucet aspirator may be connected to a long open U-tube mercury manometer to show that the pressure at the constriction when water is flowing is much less than atmospheric pressure, even though the static pressure of water in the mains is greater than atmospheric.

An atomizer or insecticide sprayer may be shown. Its principle is illustrated in simple fashion by blowing through a horizontal tube across the upper end of a second tube dipping into a beaker of water. The reduced pressure caused by the rapid passage of air across the mouth of the vertical tube causes water to rise several inches above the level in the beaker.

M-305. Venturi Meter. The rate of flow of fluids is frequently measured by the type of instrument shown in Fig. 93. A manometer is attached to the flow tube CD at A and B; the area of cross section at B is less than that at A. When fluid flows through the tube CD, the pressure at B is less than at A, as shown by the difference in level of liquid in the manometer. For air flow, water may be used in the manometer; for liquids, mercury is preferable.

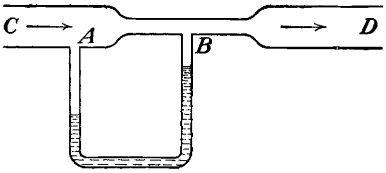

FIG. 93.—Venturi meter.

M-306. Streamline Apparatus. An apparatus for showing streamlines, somewhat difficult to construct, is commercially available. It consists of a series of projection cells, with various-shaped obstacles fitting between their parallel walls and with an outlet at the bottom. These cells may be attached in turn to a double-compartment tank, one side filled with clear, the other with colored water. Through a row of holes in each of these tanks, the liquids flow into the projection cell in a series of alternate clear and dark lines, whose shape may be observed as they bend around the obstacle.

M-307. Air Flow Made Visible. A simple and effective method of rendering air streams visible consists in producing a jet of dense fog from solid carbon dioxide and water. A few pieces of dry ice are introduced into a large flask half full of warm water. The flask is closed with a stopper through which emerges a tube of large diameter. The dry ice sublimes rapidly and forms a heavy white fog. The mouth of the exit tube may be placed in the wind stream of an electric fan so that the outrushing fog is directed along the stream. Strong illumination helps to render the flow visible even in a large room. Disks, spheres, and streamlined objects of the same cross-sectional area may be introduced into the visible stream to compare their various effects upon the flow.

M-308. Streamline Experiments. Place a lighted candle near a cylindrical obstacle, such as a large beaker or tin can. Blow the candle out by directing a blast from the lungs at the side of the cylinder diametrically opposite the candle.

M-309. A card about the size of a calling card is bent at right angles near its ends to form two legs about $\frac{3}{8}$ in. high. Stand the card on its legs, and invite a student to blow it over. Direct blowing between lecture table and card makes the card cling firmly to the table (Bernoulli effect). But if the card is placed 18 in. or 2 ft away and a person blows perpendicular to the table, the card promptly turns over. In this case the streamlines curl upward after following the table for some distance (Fig. 94).

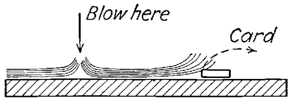

FIG. 94.—Bent calling-card turns over in upward streamlines.

M-310. Velocity Distribution in Streamline Flow. The lower half of a glass tube 1.5 in. in diameter and 12 in. long is filled with clear castor oil. The upper half is filled with castor oil colored red.[1] Some of the clear oil is now drawn off at the bottom through a tube with a pinchcock. The rate of flow of the oil is greatest on the axis of the large tube and decreases toward the

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[1] An oil-soluble dye is mixed in a small amount of carbon tetrachloride that is in turn mixed in castor oil.

walls, as is clearly evident from the shape of the interface between the clear and the colored oil.

M-311. Streamline vs. Turbulent Flow. A steel ball or a marble dropped into a tall glass graduate filled with viscous liquid (glycerin, castor oil, corn sirup, etc.) will fall slowly straight to the bottom. But if the ball is dropped into water or gasoline, it soon acquires such speed that the motion of fluid past the ball becomes turbulent, so that the ball moves in a sinuous path and strikes the side of the glass cylinder before it reaches the bottom.

A hot iron ball lowered suddenly in front of a bright pinhole source of light in a dark room shows, by shadow projection, the turbulent vortices that form behind it in the air. If the ball is moved slowly, the motion of air currents is smooth and undisturbed.

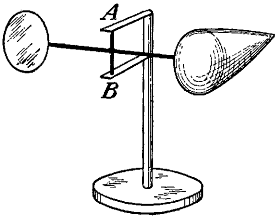

Fig. 95.—Streamlining reduces wind resistance.

M-312. Advantage of Streamline Design. A metal disk and a wooden streamlined object whose maximum area of cross section is equal to that of the disk are fastened to a stiff wire 6 in. long and balanced about a vertical wire pivot AB midway between them (Fig. 95). An air stream from an electric fan is directed at both objects. The streamlined body moves forward into the air stream.

If desired, the disk and the cigar-shaped object may be replaced by two toy automobiles, one streamlined, the other not.

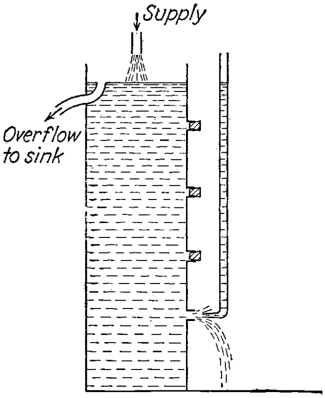

Fig. 96.—Torricelli's theorem; velocity of efflux shown by Pitot tube.

M-313. Torricelli's Theorem. A tank with several side openings allows water to stream out in parabolic paths from which the initial velocity of efflux may be computed. More simply, the velocity may be determined by a Pitot tube, a glass tube of length equal to the height of the tank and drawn down to a small nozzle extending at right angles to the length of the tube (Fig. 96). When this tube is placed in any one of the emerging streams, the pressure that it shows is (nearly) the same as the head in the tank. It is probably worth while to show that the velocity of efflux is (nearly) independent of the size of the hole, by having two holes at the same height but of different diameters. An overflow tube and constant supply of water from the mains serve to keep the level in the tank constant.

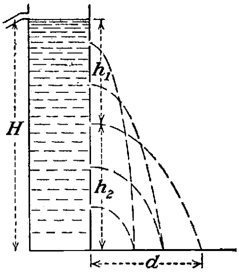

FIG. 97.—The middle jet shoots farthest.

M-314. Water Parabolas. A tall reservoir, equipped with overflow tube to give constant head, is supplied with water from the mains. Water may flow from any one of several horizontal orifices, equally spaced above the table on which the reservoir stands (Fig. 97). The (theoretical) distance *d* at which any jet strikes the table is $2\sqrt{h_1h_2}$, where h_2 is the height of the orifice above the table and h_1 is the pressure head. This distance is maximum when $h_2 = h_1$, *i.e.*, for a hole halfway up the reservoir.

MECHANICS OF GASES

M-315. Gases Have Mass. Exhaust a large glass balloon with a tight stopcock. Balance it on the scales. Let the air in and show that its weight has increased about one gram per liter.

M-316. Gases May Be Poured Like Liquids. Two 500-ml beakers are balanced on the scales. Into one of them may be poured the heavy vapor of sulfuric ether until that side of the balance is depressed. The flow of vapor is readily made visible by shadow projection (L-30). Carbon dioxide may likewise be used.

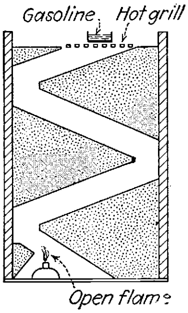

FIG. 98.—Gasoline vapor runs down the ramp and a flame strikes back.

M-317. Safety First. A teaspoonful of gasoline in an open tray is placed on a hot wire grill at the top of a "staircase" (Fig. 98). At the bottom is the open flame of a candle. In a few seconds, the heavy vapors descend the "staircase," and a flame strikes back to ignite the gasoline at the top. The front of the box representing the staircase is sealed with glass for visibility. The connection between this experiment and the all-too-frequent accidents from gasoline vapor may be emphasized.

M-318. Gas Siphon. Into a beaker on a stand about 1 ft high flows carbon dioxide from a rubber tube connected to a gas cylinder through a reducing valve. A second piece of tubing, filled with carbon dioxide, is used as a siphon to drain gas from the upper beaker into a lower one set on the table. The presence of the carbon dioxide in the lower beaker is shown by extinguishing a lighted candle set in it.

M-319. Compressibility of Air—Boyle's Law. For demonstration purposes, a large model of the Boyle's law apparatus is desirable. The scales should be graduated with broad clear lines. Both the compression chamber *C*, a $1\frac{1}{2}$-in. tube about 1 ft long, and the mercury reservoir *R* should be firmly mounted on blocks that are attached to ropes passing over pulleys and counterbalanced (Fig. 99).[1] The scales showing the pressure may be drawn on ribbons passing over spools, so that the zero of P_1 may be adjusted to the level of mercury in the compression chamber. To emphasize that it is the absolute pressure that enters Boyle's law, the left side P_1 starts from zero and gives plus and minus inches of mercury, and the right-hand scale P_2 runs from 10 to 60 in., with the local barometer reading set opposite the zero of the left scale. The stopcock in the top of the compression chamber must be carefully ground and greased to prevent leakage. Volumes are read directly from the scale *V*.

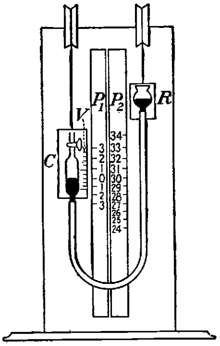

FIG. 99.—Apparatus for verifying Boyle's law.

M-320. Cartesian Diver. The diver is a test tube ($\frac{1}{2}$ by 4 in.) about two-thirds full of kerosene colored with alkannin dye (which is insoluble in water). While the finger is held over the end, the tube is inverted and dipped into water in a tall jar, and the short arm of a J-tube introduced, by which the volume of air above the kerosene may be adjusted. The tube should float with its top about 2 mm above the water surface. A little water is removed from the jar, and the top closed with a one-hole rubber stopper through which passes a glass tube attached to a length of rubber tubing. Pressure can be applied to the water surface by blowing through the rubber tube, thus causing the diver to

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[1] KREIDER, D. A., *Am. Jour. Sci.*, **32**, 329, 1911.

descend. If the adjustment is right, it will be found that there is a point about halfway down where equilibrium is indifferent —below this point, the diver will continue to sink even when the air pressure is relieved. It can be brought to the surface again by sucking through the tube. The jar should be well illuminated and backed by a white cardboard.

M-321. A small medicine or perfume vial, preferably of colored glass, inverted with the proper amount of entrapped air, in a flat bottle (*e.g.*, a quart ammonia bottle) serves as the diver. The diver may be made to ascend or descend by change of pressure as previously described (M-320). However, if the bottle is closed with a rubber stopper and the air pressure within the bottle is adjusted critically, the diver may be made to ascend or descend by squeezing the bottle. The proper adjustment may be made by moving the stopper, provided that only a little air remains in the bottle.

ATMOSPHERIC PRESSURE

M-322. Simple Demonstrations. A cavity is hollowed out in the top of a cork stopper, keeping the rim of this face smooth and intact. If the cork is pressed, cavity downward, against the bottom of a tall graduate while water or mercury is poured in, the cork will remain on the bottom.

A smooth heavy object may be lifted by pressing one or more rubber vacuum cups upon it and then lifting on the cups.

Two large test tubes are chosen, one fitting nicely into the other. The larger test tube is filled with water, the smaller one inserted a little way into the mouth of the larger, and the system inverted. Water runs out, surface tension prevents the entry of air, and atmospheric pressure raises the smaller test tube.

A glass bottle is filled with water and closed with a rubber stopper through which passes a glass tube reaching nearly to the bottom. Two questions are proposed to the class: Can the bottle be exhausted (*a*) in the upright position? (*b*) in the inverted position? The answer is given by attaching it to an aspirator (M-304) in a near-by sink. This experiment is designed to show the limitation of "suction."

M-323. Magdeburg Hemispheres. Most laboratories are equipped with the standard apparatus for repeating this historic experiment. A modern modification consists of a cylinder at least 5 in. in diameter, closed with a tightly fitting piston. The cylinder is hung by chains from a rigid support. From the piston is suspended a platform on which a student may stand or sit in a chair. Upon pumping out the cylinder with a good pump, the student is lifted. Extremely low pressures are not required provided that the diameter of the cylinder is large enough. For example, the cylinder may be replaced by a bellows 2 ft in diameter, and a student may be lifted by connecting the bellows to an ordinary vacuum cleaner.

M-324. Barometer Tube. A tall vessel or tube is filled to a depth of 80 cm with mercury. Into this is dipped a glass tube with an open stopcock at the upper end. When the mercury completely fills the inner tube, the stopcock is closed and the tube withdrawn until the mercury falls away from the top, showing the barometric height. If this tube is somewhat enlarged just beneath the stopcock, a small bladder, carefully sealed, may be placed within it, and it will expand in the Torricellian vacuum.

This setup affords the opportunity to ask a catch question. If the glass tube is supported by a spring balance, does the pull on the balance equal only the weight of the tube, since the mercury is held up by atmospheric pressure? The correct answer is demonstrated.

M-325. Pressure Due to Height. A ½-in. brass tube about 4 ft long is closed at each end. Two fine holes of equal size are bored near the ends. At the center of the tube is soldered a ¼-in. tube for connection to the gas supply. When the tube is held horizontally, the two flames issuing from the holes are the same height, but as the tube is rotated to the vertical position, the flame at the top becomes longer than the one at the bottom, because the gas used is lighter than air.

M-326. Collapse of Tin Can. A large varnish or alcohol can is heated with a small amount of water in it. When the air is thoroughly driven out by steam from the boiling water, the can is tightly corked and the heat turned off simultaneously. Upon cooling, the can collapses under atmospheric pressure; the collapse may be hastened by cooling the can with water (H-77).

A more positive procedure is to fit the can with a valve, which may in turn be connected directly to the air pump. The exhaustion is carried only to the point where the can crumples slightly. Then the can is removed from the pump, air is let in to atmospheric pressure, and the valve is closed. The can is now placed beneath a large bell jar on the plate of the vacuum pump, and the space around it is exhausted. The can is blown outward by the entrapped air.

M-327. Buoyant Effect of Air. A large cork stopper or hollow sealed glass ball is balanced against a brass weight on a simple balance placed under a bell jar. When the air is pumped out, the cork falls, showing that the air buoyed up the cork more than it did the brass weight (M-315).

M-328. Gas-filled Balloons. Balloons may be filled to any desired pressure from the gas mains by first letting the gas from the main enter an auxiliary balloon or rubber bulb. The valve to the main is closed and another to the balloon is opened so that the gas may be forced from the auxiliary bulb into the balloon. This process is repeated cyclically until the desired inflation is obtained. Using hydrogen and illuminating gas at equal pressures in identical balloons, the difference in lifting power may be shown by the load each will carry or by their relative rates of ascent without load.