Understanding Quantum Mechanics

- Colleagues:
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- Principal Critics:
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- Books:
  - by R. Omnès:
    The Interpretation of Quantum Mechanics (1994)
    Understanding Quantum Mechanics (1999)
  - by R. B. Griffiths
    Consistent Quantum Theory (2002)
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Quantum Difficulties

- Students find quantum mechanics difficult because of:
  - Unfamiliar mathematics
  - Quantum world different from familiar classical world
  - Unresolved conceptual issues
  - – Unclear to teacher ⇒ Opaque to student

- Why are there unresolved conceptual issues?
  - They occur with any new subject
  - Success of quantum mechanics
  - Difficulties assigned to "Quantum Foundations"
  - – Discipline not held in high regard

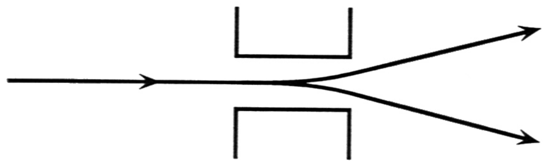
- Einstein, Schrödinger, Wigner, Feynman
  - Famous for their work on quantum mechanics
  - Admitted they did not understand it!
  - Feynman: Nobody understands quantum mechanics

- Recent (last 20 years) progress:
  - Quantum incompatibility
  - Quantum measurements
  - Quantum probabilities

Incompatibility I. Stern-Gerlach

- What key feature separates quantum, classical physics?
  - Bohr: Complementarity
  - Heisenberg: Uncertainty
  - *Quantum incompatibility* includes them both
  - Is a precise idea, based upon Hilbert space

- Stern-Gerlach experiment (1922)



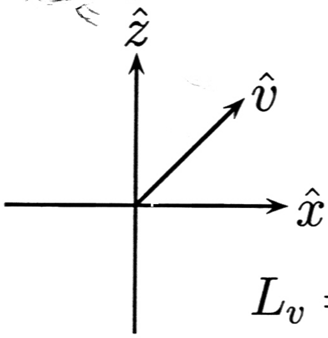
  - Silver atoms in ground state (S=1/2)
  - Spin 1/2 atom can have only two values of S_z
  - But what is special about S_z?

Quantized Angular Momentum I

- Classical angular momentum vector $\vec{L}$ $\quad (L_x, L_y, L_z)$
  - $L_w = \hat{w} \cdot \vec{L}$ $\quad$ ($\hat{w}$ – unit vector in direction w)

is the component of angular momentum in direction w

  - Example:



$(1/\sqrt{2}, 0, 1/\sqrt{2})$

$$L_v = (L_x + L_z)/\sqrt{2}$$

- Spin half particle:
  - For *any* w, $S_w = +1/2$ or $-1/2$ (in units of $\hbar$)
  - $S_z = +1/2$ or $-1/2$
  - $S_x = +1/2$ or $-1/2$
  - So if $S_v = (S_x + S_z)/\sqrt{2}$ then

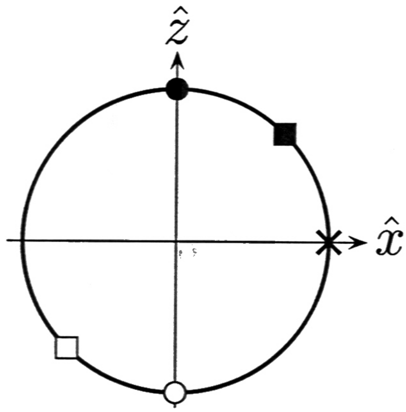
$$S_v = 1/\sqrt{2} \text{ or } 0 \text{ or } -1/\sqrt{2}$$

  - This is inconsistent with $S_v = +1/2$ or $-1/2$
  - What has gone wrong?

Quantized Angular Momentum II

- What is wrong with $S_v = (S_x + S_z)/\sqrt{2}$?
  - ○ Quantum, classical systems are described by *different mathematics*

- Spin 1/2 requires 2-dimensional Hilbert space
  - ○ Bloch sphere picture:
  - – Each point on sphere $\leftrightarrow$ quantum state



- ● $S_z = 1/2$
- ■ $S_v = 1/2$
- × $S_x = 1/2$
- □ $S_v = -1/2$
- ○ $S_z = -1/2$

- Which state describes $S_x = +1/2$ AND $S_z = +1/2$?
  - ○ There is NO such state in the Hilbert space!

Incompatibility II

- Incompatibility illustrated using spin half:
  - $S_z = +1/2$, $S_z = -1/2$ are mutually exclusive; one or the other is true
    - Check this using measurements
  - $S_x = +1/2$, $S_x = -1/2$ are mutually exclusive
  - $S_z = +1/2$ AND $S_x = -1/2$ is *meaningless*: no counterpart in the Hilbert space

- Contrast with the classical world:
  - Tony Blair IS/IS NOT Prime Minister of UK
    - Mutually exclusive; one and only one true
  - Al Gore IS/IS NOT President of USA
  - (Blair IS Prime Minister) AND (Gore IS President): meaningful but false

- No quantum state $\leftrightarrow$ $S_z = +1/2$ AND $S_x = -1/2$
  - Does this mean that it is always false?
  - If always false, its negation is always true
  - Is $S_z = -1/2$ OR $S_x = +1/2$ always true?
  - But what if $S_z = +1/2$?

Incompatibility III

- Incompatible quantum descriptions cannot be combined
  - Consequence of Hilbert space mathematics
  - Is there any way around this?

- Alter the rules of logic
  - Birkhoff, von Neumann proposal: quantum logic
  - Does it solve the paradoxes?

- Change the mathematics
  - Add (hidden) variables to Hilbert space
  - Bohmian mechanics an example, but its long-range influences conflict with relativity
  - Inevitable problem for hidden variables (Bell)

- Don't talk about microscopic systems:
  - Only results of measurements are meaningful
  - Great Smoky Dragon (John Wheeler)

Measurements

- J. von Neumann (Mathematatische Grundlagen . . . 1932)
  - Unitary time development of isolated system
  - Interaction with measurement apparatus
  - Collapse of the wave function
- Objections:
  - "Collapse" seems odd in a physical theory
  - Applies only to *nondestructive* measurements
  - What is special about measurements?
  - – Measurement apparatus composed of atoms, etc.
  - Include apparatus in unitary time development?
  - – theory is inconsistent!
- Conventional measurement theory is
  - Awkward, unnatural
  - Not applicable to real laboratory measurements
  - Internally inconsistent
  - Not what physicists use in practice
- Why is it still in the textbooks?
  - Introduces probabilities into quantum theory
- Can one do better?

Probabilities I. Sample Spaces

• Sample space: Mutually exclusive possibilities, one and only one of which occurs in a given experiment

∘ Example: Heads, tails for coin

∘ Example: $s = 1, 2, 3, 4, 5, 6$ for die

• Quantum sample space for spin half:

∘ $S_z = +1/2$, $S_z = -1/2$ are like H, T for coin

– One and only one is true

– Measurement (Stern-Gerlach) tells you which

∘ $S_x = +1/2$, $S_x = -1/2$ also a sample space

• S_z and S_x sample spaces *incompatible*

∘ Combining them makes no sense

∘ Cannot ask if $S_z = +1/2$ OR $S_x = +1/2$:

– they are *not* mutually exclusive possibilities

• "Cannot measure both S_z and S_x"

∘ This is true, but misses the point:

∘ What does not exist cannot be measured!

Probabilities II. Born Rule

- Born rule for probabilities:
  - ◦ Initial $|\psi_0\rangle$ at $t_0 \to |\psi_1\rangle$ at t_1 using Schr eqn
  - ◦ $\Pr(\phi)$ = probability of $|\phi\rangle$ at t_1 is $|\langle\phi|\psi_1\rangle|^2$
  - ◦ Must choose a sample space at t_1:
  - – $|\phi\rangle$ part of orthonormal basis $\{|\phi_k\rangle\}$
  - ◦ Cannot combine incompatible sample spaces

- Measurements:
  - ◦ Measurement reveals prior state of system
  - ◦ Example: SG spin measurement of S_z
  - – Apparatus outcome + means S_z was earlier $+1/2$

- Contrast with von Neumann approach in which:
  - ◦ Born rule → probability of *measurement outcome*
  - ◦ System state before measurement is *unknown*

- Extended Born rule for histories
  - ◦ Born rule limited to 2 times
  - ◦ Extending it requires:
  - – Appropriate sample space
  - – Consistency conditions

Quantum Paradoxes

- Partial list of quantum paradoxes
  - Double slit ◦ Schrödinger's cat
  - Einstein-Podolsky-Rosen ◦ Bell-Kochen-Specker
  - Hardy ◦ 3 boxes (Aharonov, Vaidman)

- Tame paradox: surprising, but can be understood using consistent theory; no logical paradox
  - Example: twin paradox in relativity

- Schrödinger cat, Bell-Kochen-Specker
  - Quantum theory allows alternative descriptions
  - Precise formulation of incompatibility

- Einstein-Podolsky-Rosen, Hardy
  - Dependent (contextual) properties
  - Alternative descriptions
  - Consistent counterfactuals
  - There are NO long-range influences

- Double slit, three boxes
  - Quantum histories, consistency conditions
  - Alternative descriptions

Summary

- Sources of quantum conceptual difficulties:
  - Unclear ideas about incompatibility
  - Interpretation of theory, probabilities based on concept of measurement
- Solution to conceptual difficulties:
  - Hilbert space math as guide to interpretation
    - In particular, incompatibility
  - Probabilities intrinsic to dynamics and NOT based on measurements
- Consequences:
  - Precise rules for quantum reasoning
  - Taming of paradoxes
  - Measurements are ordinary quantum processes
  - No wave function collapse
  - No long-range, superluminal influences:
    - Removes any conflict with relativity
- Can we get the new ideas into the textbooks?